

K]Y k]f F i gg`Ub X ZU`gWX j YfghUb X Y b \ UV Y b

=b Figg`UbX'[UV'Yg'_Y]bY'GW\bAddW\Yb

9bXY '>Jbi Uf 'gWX f]YV ']WX 'Y]bYb '5fh]_Y `a]h XYa 'H]hY `GWX bÀdd WX Yb 'U[X ']b 'F i gg`UbX ž'Y]b 'GUhnž XYb ']WX 'Y]bYb 'A cbUh gdÀhYf 'b]WX h'a Y\f '[YfbY 'UVh]ddY "'8Yf '5fh]_Y `k]Yg XUfUi Z'\]bž XUgg 'Y]bY '? ca V]bUh]cb 'Ui g '\c\Yb '9bYf[]YdfY]gYb 'i bX GdUbbi b[Yb 'Ub 'XYf'; fYbnY 'ni f'I_fU]bY 'fi gg]gWX Y '5_h]Yb 'gY\f

I b\YX[YX'VYZUggh'g]W\'b]W\ha]h'XYf'6Yk Yfhi b[
[Ycdc`]h]gW\Yf'F]g]_Yb"'5VYf'k Ybb'g]W\'X]Y'G]hi Uh]cb'Ub
XYf'; fYbnY'UV_Ø\`hž'k]fX'Ya UbX']b'Fi gg`UbX'j]Y`'; Y`X
j YfX]YbYb"

 $8] YgY : 5i ggU[Y'k] fX'nk Y] a U``UV[Yg] W. Yfhž'YfghYbgž'] bXYa '] W. gU[Yž'XUgg'] W. 'VY] a 'fY`Yj UbhYghYb'F]g]_c'_Y] bY : 5\bib['\UhhYž k cj cb'] W. 'fYXYž'i bX'nk Y] hYbgž'] bXYa '] W. 'XYb'hfYi Yb': fYi bX'XYg \(\sigma i \text{fbU} \) ghYbž'Y] bY : 6YX] b[i b[ž'Y] bgYhnY'': 5VYf'hfchnXYa 'g] Y\h'XUg GhØ \(W_' \) a 'B UW\\] bY] b'gW. fY \(W_' \) W. 'Ui g'': 5V[YgY\Yb'j cb'XYb 6YX] b[i b[Yb'bU\a'XUg'GhØ \(W_' \) X] Y'F] g]_cgY] hY'XYg'@YX[Yfg'b] W. h] a '9bhZYfbhYghYb'Yfbgh'[Ybi['']$

 $A Y]b \cdot 5fh]_Y \cdot \ \ Uh'X]Y \cdot 5f[i \ a \ YbhY \cdot ZØf'fi \ gg]gWXY \cdot 5_h]Yb \cdot k \]Y \cdot Zc \cdot [h \ WX UfU_hYf]q]Yfh$

8]Y`"`"\8Ugʻ5f[ia YbhʻXYfʻfi gg]gW\Ybʻ6i``YbʻVYghY\hʻUi g nk Y]`HY]`Yb"\"\9fghYbg. `A cg_Ui `]ghʻYfbgh\UZhii a `gY]bY Y][YbYʻG]W\Yf\Y]hʻVYgcf[hii bX`\Uhʻk YXYfʻ]bbYbdc`]h]gW\Y I bhYfghØhni b['bcW\`[YcghfUhY[]gW\Y*bhYfYggYb`Ub`Y]bYf 5bbYI]cb`i_fU]b]gW\Yb`HYff]hcf]i a g"`K Ybb`X]Y`I G5`i bX X]Y`BUhc`Ygʻ_Ø\`Ybž']bXYa `g]Y`Ni g]W\Yfi b[Yb`]b`6Yni [Ui Z'F U_YhYb'i bX'Y]bY'DUi gY']b'XYb 9I dUbg]cbgVYghfYVi b[Yb'XYf'BUhc'UbV]YhYbž'k ØfXY'Y]bY 8YYq_U`Uh]cb'Zc`[Yb"'

Nk Y]hYbg. 'GY`Vgh'k Ybb 'Yg`gW\`]a a Yf`k]fXž`g]bX
:]bUbngUb_h]cbYb '[Y[Yb 'Fi gg`UbX'k U\fgW\Y]b`]W\ 'ni
gW\k UW\ž`i a 'gY]bY'K]fhgW\UZh'ni 'VYXfc\Ybž'Yg`gY]'XYbbž
g]Y`\]bXYfb 'XUg`@UbX'XUfUbž'gY]b'^2 ``i bX'; Ug`]a '5i g`UbX
ni 'j Yf_Ui ZYb' 'k Ug'9i fcdU'[YbUi gc'cXYf'j]Y``Y]W\h'a Y\f
gW\UXYb'k ØfXY'U`g'Fi gg`UbX'"

5f[ia Ybh'Y]bg'ØVYfgW\AhnhY'K `UX]a]f'Di h]bg'Gcf[Y'ia]bbYbdc`]h]gW\Y'l bhYfghØhni b['ibX'[YcghfUhY[]gW\Y'=bhYfYggYbžk]Y'g]Y'\Yf_Òa a`]W\YfkY]gY'XYZ]b]Yfh'k YfXYb"'8 Ug'GW\Y]hYfb'j cb5f[ia Ybh'nk Y]'Zc`[hY'X]fY_h"'5`g'Di h]b'XYb'cZZYbYb'?f]Y['k\A\`hYžkifXY'X]Y'G_Ydg]g'ØVYf'X]Y'A UW\h'j cb'GUb_h]cbYb'j YfZA`gW\h"Figg`UbXg'8Yj]gYbfYgYfj Yb'kifXYb'XifW\"GUb_h]cbYb'[Y[Yb'X]YNYbhfU`VUb_'YZZY_h]j'nYfghÒfhžibX'gY]b'VfY]hYfYg'6Ub_YbgmghYa`]Y[h'VYfY]hg'Ui Z'XYb'?b]Yb"

8Yf'YfghY'Di b_hž'XYf'\]Yf'Ub[Ya Yf_h'k YfXYb'a i gg' ni f?\UfghY'\i b[\z'\b]W\h'ni f'A]\XYfi b[\cong]ghž'XUgg'gY\f'\ØVYffUgW\YbXY\8]b[Y'dUgg]YfYb"'K Ybb'Ya UbX'gU[h' \(\text{-\text{W}} \cong V]b'a]f'ni '-)'DfcnYbh g]W\Yfz'XUgg'L 'i bX'XUbb'B]W\h'L'j cf_ca a hž'_Ubb'X]Y

K U\fgW\Y]b\]W_Y]hggW\\Ahni b['XYbbcW\'_cffY_h'[Yk YgYb'gY]b" K]Y'XYf'DgmW\\c'\c[Y'D\]\]d'HYh\c\W_'VYhcbhz'\Aggh'g]W\'X]Yg'bi f

VYi fhY]\Ybz']bXYa 'a Ub'df\Ozhz'cV'X]Y

K U\fg\W\Y]b\]W_Y]hgg\W\Ahni b[Yb'Y]bYf'VYgh]a a hYb'DYfgcb']a

@Ui ZY'XYf'NY]h'i b[YZ\A\f'f]W\h]['g]bX'''XUg'\Y]\\h'z'X]Y'8]b[Y\z'\X]Y

g]Y'gU[Yb\z'\UVYb'VY]gd]Y\gk Y]gY'Y]bY'+)!dfcnYbh][Y

K U\fg\W\Y]b\]W_Y]h\z'XUgg'g]Y'Y]bhfYhYb'\z'dUgg]YfYb'Yhk U'+)

DfcnYbh'XYf'NY]h''

5VYf`XUg``Àggh`k YXYf`a]W\`bcW\`]f[YbXYa UbX`UbXYfYb`XUj cb UVž`XUfUb`ni `nk Y]ZY`bž`XUgg`Y]bY`fY[Y`fYW\hY`\#j Ug]cb`ghUhhZ]bXYb

k ØfXY""=\\\ 'XYb_YžY]b"HY]\'a Y]bYg"DfcV\'Ya g'i bX'j]Y\'Yf"UbXYfYf @Yi hY'k Ufž'a]h'k Y\'\\Yb"9I dYfhYb'k]f'[Ygdfc\\\Yb'\UVYb"5\\Y\ @Yi hYž'a]h'XYbYb']\\\ [Ygdfc\\\Yb'\UVYž'k UfYb'GhfUhY[Yb'i bX DcfhZc\]ca UbU[Yfž'X]Y'Fi gg\\UbX'i bX'fi gg]g\\\Y'5_h]Yb'gY]h\\Ub[Ya j YfZc\[Yb"5VYf'k Ybb'a Ub'\ØVYf'F]g]_Yb'bU\\XYb_hž']gh'Yg [YZ\\f\]\\Z'X]Y\\g'\Sfh'j cb'\Dg\Yb'\ni 'j Yf\\UggYb'' : c\[YbXYg'gU[hY'a]f'Y]b'\Dj Yghcfž'XYf']b'Fi gg\`UbX'\YVh

9]b 'Ok cfghl WUgY 'gWW b Uf]c O]gh'Y]b Y '\(\) j Ug]cb 'XYf'I _fU]b Y ž VY]'XYf'fi gg]gWX Y 'DUbnYf'X]Y'; fYbnY 'ØVYfei YfYb'i bX'Y]b [fc * Yg'HYff]hcf]i a 'Y]bbY\a Yb" '8 Ug']gh'gY\fž'gY\f i bk U\fgWXY]b`]WX. '8 UZØf'[]Vh'Yg']a 'fi gg]gWX Yb'J c`_'_Y]bY I bhYfghØhni b["'9g'[UV'I bhYfghØhni b['ZØf'X]Y'5bbYI]cb XYf'?f]a "'J cb'a Y]bYb': fYi bXYb']b'Fi gg`UbX'\UVYb j]Y``Y]WXh'%\$!%) 'DfcnYbh'i _fU]b]gWXY'DÀggY' 'Yg'[]Vh'j]Y`Y ZfYi bXgWX UZh`]WXY'6Yn]Y\i b[Ybž'a]h'XYf': Ua]`]Y'Ub'XYa Y]bYb'cXYf'UbXYfYb'Cfh"'9g']gh'k]Y']b'Bcfk Y[Yb'i bX GWXk YXYb"'5`gc'k ØfXY']WX'X]YgYg'GnYbUf]c'UVgWXfY]VYb"

 $\label{thm:linear:li$

#D'XYb'`YhnhYb'HU[Yb'\UVYb'j]Y`Y'5i » Ybdc`]h]_Yf'XUfØVYf
[YgdfcWXYbž'k]Y'g]WX'Di h]b'j YfÀbXYfh'\Uhii bX']ffUh]cbU`Yf
[Yk cfXYb']gh' 'XUgg'Yf' b]WXh'XYf'Di h]b']ghž'XYb'k]f'_UbbhYb "
A U['gY]bž'XUgg'Di h]b'Yhnh'f]g]_cZfYi X][Yf']gh''5VYf'X]Yg'Ui Z
dgmWX]gWXY'9f_fUb_i b[Ybž'7cj]X! =gc`Uh]cb'cXYf'XUg'5`hYf
ni fØW_ni ZØ\fYbž']gh'Ui WX'Y]b'K Y['ZØf'ØVYffUgWXhY'A YbgWXYbž
]\fY'Y][YbY'_c[b]h]j Y'8]ggcbUbn'ni 'fYXi n]YfYb''K]Y'HYh'cW_'Yg
a]f']b'Y]bYf'9!A U]`a]hhY]`hYž'Yf`Ui Vh'X]YgY'NY]`Y'ØVYffUgWXhYb

9IdYfhYb'ni 'gU[Yb.' Bibž'Yf'\ÀhhY'[YhUbž'k Ug']W\'Yfk UfhYh'\UhhYžk Ybb'Yf'b]W\h'Ub'II 'Yf_fUb_h'k ÀfY "

<]Yf'[]Vh'Yg'a Y]bYf'A Y]bi b['bUW\'bcW\'Y]b'k Y]hYfYg'DfcV'Ya "
GcVU'X'k]f'Y]bYa '9f[YVb]g'Y]bY'[Yf]b[Y'K U\fgW\Y]b']W_Y]h
ni cfXbYbž\\OfYb'k]f'cZh'Ui Zž']bhYbg]j 'ØVYf'X]Y': c`[Yb'X]YgYg
9f[YVb]ggYg'bUW\ni XYb_Yb"'\dangle X]YgYa ': U``'gU[hY'Y]bYf'XYf
GhfUhY[Ybž'a]h'XYbYb']W\'[YgdfcW\Yb'\UVYž'XUgg' Yg'_Ui a
dUgg]YfYb'_Ubbž'XUgg'Yhk Ug'GW\`]a a YfYg'dUgg]Yfh'U`g'XUgž'k Ug
VYfY]hg'Y]b[YdfY]gh']gh ' 'Y]b'j YfhfUi hYf'FYZfU]bž'k Ybb'j YfVYi `hY
J Yfa O[Ybgk YfhY'Ub[YVchYb'k YfXYb"'Bi bž'Ui g'G]W\h
Ui g'AbX]gW\Yf'\dangle j YghcfYb'g]Y\h'Yg'`Ub[gUa 'XUbUW\'Ui gž'XUgg
j]Y`Y'fi gg]gW\Y'JYfa O[Ybgk YfhY'k Yb]['cXYf'[Uf'b]W\hg'k Yfh'g]bXž
k Ybb'5b'Y]\Yb'Ui gZU``Yb'i bX'8]j]XYbXYb'[YgdYffh'k YfXYb"
GY'Vgh'k Ybb'G]Y'a]h'Y]bYa 'k]bn][Yb'DfcnYbhgUhn'\dangle fYg'DcfhZc`]cg
Ui Z'\cW\f]g_UbhY'fi gg]gW\Y'JYfa O[Ybgk YfhY'a]h\\c\Yb'FYbX]hYb
[YghcW\Yfh \\UVYbž'hi h'Y]b'K]dYci h'k Y\"'<UVY']W\'[Ybi [XUfØVYf'bUW\[YXUW\h3</pre>

9]b``YhnhYf`; YXUb_Y"`K Ug`X]YgY`K cWXY`dUgg]Yfh`]ghž`\Uh`a]WX
YfbYi h'ØVYf`XUg`b]WXh`]bj Ygh]YfVUfY`F]g]_c`bUWXXYb_Yb``UggYb"
;]Vh`Yg`F]g]_Ybž`X]Y`Y]bZUWX`ni `_ca d`Yl `g]bXž`i a `g]Y`a]h`XYf
DfÀn]g]cb`ni `UbU`mg]YfYbž`X]Y`\Dj Ygh]h]cbYb`YfZcfXYfb3`<ÀhhYb`k]f
j cf\Yf`k]ggYb`a ØggYbž`XUgg`XYf`a]`]hÀf]gWXY`?cbZ]_h`]b
CghYi fcdU`Y]bYf`XUj cb`k Uf3`;]Vh`Yg`XYfnY]h`UbXYfYž`UbU`c[Yž
b]WXh`]bj Ygh]YfVUfY`F]g]_Yb`Ui Z`XYf`K Y`h3`K UffYb`6i ZZYhh`\Uh`Y]b
ni `\UfhYg `HUV`Yhh`Ui Z`gY]bYa `GWXfY]Vh]gWXž`bYVYb`XYb`ØV`]WXYb
fY]b !`i bX` fUi g !?ÒfVYbž`k c`X]YgY`F]g]_Yb``]Y[Yb"`A]WX`k ØfXY
]bhYfYgg]YfYbž`k Ug`X]Y`@YgYf`XYfnY]h`]b`X]YgY`?UhY[cf]Y
Y]bcfXbYb"

: $fY]\Y]hg[Y`X`Z`]Y\h`Ui g`Fi gg`Ub X$

 $8 \ Yf' DfY]g'j \ cb'6]hWc]b'gh]Y['Ua'A \ cbhU[']b'X]Y'< O\Y'i b X'gh]Y[']bbYf\U`V'k Yb]['Yf'Ghi b XYb'i a'''$$$'`"'5VYf']['bcf]YfY'XUg'''8]Y]bhYfYggUbhYfY'BUW\f]W\h'k Uf'Y]b'5bgh]Y['XYf'Ui Z'Fi VY``Ui hYbXYb'6]hWc]b!? Ai ZYfZ'XU'X]Y'K A\fi b['k Y]hYf'UVghØfnhY''@Ub['A\f]['Y'6]hWc]bYf'hf]i a d\]YfhYb'''<]Yf']gh'Y]b.$

 $\begin{tabular}{ll} K & $[f'_cbbhYb'b]Wh'XYb'NY]hdib_hcXYf'X]Y'5fh'ibX'KY]gY \\ k & $A'Ybž']b'XYf'ibgYfY'_Y]bY'& Xighf]Y'&VYf'BUWh \\ & [Ycdc]h]gWh'_f]h]gWh'kifXYž'UVYf'Yg'Y]Y[h'Ub'ibg'' \\ \end{tabular}$

8]Y'BUhgYW'i bX'BYcWcb!HmdYb'k YfXYb'X]Y'HUhgUWXY
b]W\ha O[Yb\z\u\g\Gub_h]cbYb'i bj YfV\@a h'g]bX'i bX'U\g
\delta\ghfi a Ybh\v\u\x\@v\Yf\c\h'g]bX\z\u\Yf\k Ybb'X]Y'K U\\
nk]g\w\Yb'a cbYh\Af\Yf'Gci j Yf\\b\b]h\h'Ui Z'ghUUh\]\w\Yf'i bX
]bX]j]Xi Y\"\Yf'9\YbY'\V\Ygh\Y\h'i bX\\\\Y\Yf\Y]bY'j cb'\Y]bYf
Y]bn][Yb'FY[]\Yfi b['_cbhfc\"]\Yfh\Y'; Y\XXUh\Yb\V\Ub_
j Yfk YbX\Yh \z'k \@f\X\Y']\w\\k]gg\Yb\z'k c']\w\'gh\Y\\"

K]Y'k]f``YhnhY'K cW\Y'ZYghghY``hYbž'[Yk]bbh'Y]bY': fYYXca
A cbYm !J]g]cb'j cb'6]hWc]b'Ub'?fUZhž'X]Y'j Yfgdf]W\hž'X]Y'ghUUh`]W\Y
?cbhfc``Y'ni ii a [Y\Yb"'8]Y'I _fU]bY'gUa a Y`h'A]``]cbYb'j cb
7fck XZi bX]b[!6]hWc]bg'fl bhYf'UbXYfYa Ł"'8]Y'_UbUX]gW\Yb
Hfi W_Yf!8Ya cbghfUbhYb'gW\`cggYb'g]W\']\a 'Ubž'bUW\XYa 'ChhUk U
?fmdhc!; Y`XVÒfgYb']a 'F U\a Yb'gY]bYg'BchghUbXg[YgYhnYg
gUb_h]cb]Yfh'\UhhY"'i bg'i bX'Vf]h]gW\Y'?cbgYfj Uh]j Y'dY]hgW\Yb'Yg
Ui q"

⇒W. ZcfXYfY 'U``Y'[fc» Yb'? fmdhc!6ÒfgYb'Ui Zž'5XfYggYb
fi gg]gW\Yf'6Ybi hnYf'ni 'V`cW_]YfYb"'9g']gh'YbhgW\Y]XYbXž
b]W\h'bi f'X]Y'a]h'fi gg]gW\Yb'i bX'k Y]» fi gg]gW\Yb

Dc`]h]_Yfb'j Yf_bØdZhYb'5XfYggYb'Y]bni Zf]YfYbž'gcbXYfb Ui WX'bcfa U`Y'6Ybi hnYf'ni 'qUVch]YfYb"

 $G]Y'_ObbYb'X]Y'6]bXib['`]Yf'gY\Yb.'8Yf'JYfgiW\ž'; Y`X']b'X]Y\\I_fU]bY'Y]bni`UXYbž'c\bY'Yg'Uig'Figg`UbX'Z]Y\Yb'ni``UggYb'': Øfgc`W\Y'8]g_f]a]b]Yfib[']gh'XUg'hfUX]h]cbY``Y':]bUbngmghYa'VYggYf[YfØghYhž'k]Y'GUb_h]cbYb'nY][Yb''$

 $5VYf??fmdhc?gc``; Y`X`ZfY]`Z]Y*Yb``UggYb"`8Ug': Y`Yb`ghUUh`]WXYf?cbhfc``Y`]gh'b]WXh']a a Yf'gWX`YWXhž'Vf]b[h'UVYf'`\UfhY?ca dfca]ggY`a]h'g]WX"'=gh'XUg'K YfZYb'j cb'8]gg]XYbhYb'Y]bYZ]bUbn]Y``Y`6cY'k Yfhž'Y]b'i bX]WXhYfYg'GUb_h]cbgfY[]a Y`[Y[YbFi gg`UbX3'9g'VfUi h'g]WX'Y]b'Z]YgYf'dc`]h]gWXYf'?Ua dZi a '?fmdhc ni gUa a Yb"'fl9h\Ub'K i \L$

9]bY'[i hY'@Y_hØfY

9]b'Ui g[YnY]W\bYhYg'Dc`]h]Wc! = bhYfj]Yk 'a]h':]cbU'<]``'ni a 'H\Ya U Di h]bg'5a V]h]cbYb"

EiY``Y.:]bUbW]U``H]a Yg

8 YhU]`g

6Ygi WXYb G]Y i bg Ui Z _fmdhc!bYk g"Uh