

K Ug'k]f'gY\Ybž'k Ybb'k]f'FY]gYZchc[fUZ]Y VYhfUWXhYb

K Y '`cj Y 'h\Y '`cc_'UbX 'ZYY `cZ'h\Y '5dd`Y 'K UhW\"5g'k Y
a Ybh]cbYX 'UVcj Yž']h `cc_g'U'V]h `]_Y 'U'g\fi b_!Xck b'j Yfg]cb
cZ'h\Y'cf][]bU``]D\cbYž'UbX ']h 'Ua d/, &%+/g'fYUggi f]b[`m
fcVi gh '/ Ua d/, &%%/'UZhYf'U`a cgh'U'mYUf'k]h\'h\Y'K UhW\ž
h\YfY'bc'gWfUhW\Yg'cb'h\Y'VcXmcf'gWfYYbž'U`h\ci [\'h\Y
Vf][\hYf'cZ'ci f'hk c'Gdcfh'6UbX'ghfUdg']g'ghUfh]b['hc'`cc_'U
V]h'[fi VVm''5dd`Y'XcYgb/ Ua d/, &%+/h'fYWca a YbX
Xi b_]b['mci f'5dd`Y'K UhW\Z]fgh![Yb']b'k UhYf''K \]`Y'h\Y
k UhW\']g'fUhYX'Ug'k UhYf!fYg]ghUbh'hc'h\Y'-DL + 'ghUbXUfXž
k\]W\'g\ci `X'a YUb']h'k]``gi fj]j Y']b'k UhYf'i d'hc'U'XYdh\'cZ
%'a YhfY'Zcf'i d'hc'/ Ua d\Y\`]d/

5dd`Y'K UhWX fYj]Yk. 8Yg][b

 $: cf'U'g`][\h`m'a\ cfY'cd]b]cbUhYX'j\ Yfg]cb'cZ'h\Y'j\]XYc'fYj\]Yk\\ UVcj\ Yž'\YfY\ g'h\Y'a\ U`Y'\U`Z'cZ'h\Y'$ *A\ UWk\ cf`X'* $hYUa\ 'Uf[i]b['fUZhYf\ Zcif'a\ cbh\g'k]h\'h\]g'XYj\]WYŁ'UVcih'k\Yh\Yf'h\Y'5dd`Y'K\ UhW\']g\ U'[fYUh'cf'U'hYff]V`Y'dfcXi\Wh$

 $K Y j Y "gYYb "`chg"cZ"Z]hbYgg"hfUW_Yfg"cj Yf"h\ Y "mYUfgž"UbX"h\ Ymj Y hmd]WU"`m"ghfi W_"i g"Ug"dfYhhm"Zcfa i `U]W"d`Ugh]W_m"k f]ghVUbXg"k]h\$

`]hh`Y`ZUg\]cb`UddYU`"'CbY`UWh]j]hm`hfUW_Yf`VfUbX`hf]YX`hc`Wcbj]bW\ig`h\Uh`h\Y]f`UWh]j]hm`hfUW_Yf`k Ug`XYg][bYX`hc`UddYU``hc`U`ZUg\]cb!
WcbgW]ci g`k ca Ub/`h\Ym'Yj Yb`h\ci [\h`h\Uh`k ca Yb`k ci `X`k YUf`]h
Ufci bX`h\Y]f`bYW_``]_Y`U`bYW_`UWY"'6i h`Uh`h\Y`YbX`cZ`h\Y`XUmž`]h
k Ugb h`Yk Y``Yfm"'BcbY`cZ`h\Y`Z]hbYgg`hfUW_Yfg`cb`h\Y`a Uf_Yh`UfY"

#ng'U'g]a]`Uf'ghcfm'k]h\ 'ga Ufhk UhW\Yg"'Gi fYž'cj Yf'h\Y'dUgh'mYUf h\Ymj Y'VYWca Y'a cfY'UbX'a cfY'dcdi `Uf'k]h\ '[i mg'`cc_]b['Zcf h\Y'`UhYgh'hYW\'[UX[Yhž'Vi h'h\Ym'Xcb h'UddYU`'hc'Yj YfmcbY"'CbY a U'cf']ggi Y']g'h\Uh'a cgh'ga Ufhk UhW\Yg'UfY'XYg][bYX'Zcf'a Yb" H\Ym'k ci `Xb h'g]h'Wca ZcfhUV`m'cb'U'ga U``Yf'k f]gh"

5dd`Y'K UhWX fYj]Yk . 8]a Ybg]cbg

 $H\YfY'UfY'hk\ c'g]nYg'cZ'k\ UhW\.'h\Y''\ ,\ a\ a\ 'a\ cXY``fk\ \]W\ 'UWhi\ U``m\ a\ YUgi\ fYg''\ ,\ "*'Vm'' ' "' 'a\ a\ Ł'UbX'h\Y'(\ \&a\ a\ 'a\ cXY``fk\ \]W\ a\ YUgi\ fYg'(\ \&'Vm''\) "- 'a\ a\ Ł''.6ch\'\Uj\ Y'U'h\]W_bYgg'cZ'%$")\ a\ a\ "$

- ', a a 'a cXY`.'', "*'|'''' '| '%\$") a a
- (&a a 'a cXY`.'(&"\$'l'')"-'l'%\$")aa

< YfY g`\ck 'U'', a a '5dd`Y'K UhW\``cc_g'cb'?UfYb g'k f]gh.</pre>

5dd`Y'K UhWX fYj]Yk. 6i]`X'eiU`]hm

GdYU_]b['cZ'h\Y'a UhYf]U`g'h\Y'k UhW\']g'a UXY'Zfca ž'h\YfY'UfY h\fYY'cdh]cbg. 'U`i a]b]i a 'Zcf'h\Y'K UhW\'Gdcfhž'ghU]b`Ygg'ghYY`'Zcf h\Y'K UhW\'ž'UbX'%, !WUfUh'[c`X'Zcf'h\Y'K UhW\'9X]h]cb"'H\Y'K UhW\ UbX'K UhW\'9X]h]cb'Wca Y'k]h\'gUdd\]fY'gWfYYbgž'h\Y'Gdcfh j Yfg]cb'k]h\']cb!! '[`Ugg"

 $\label{thm:cc_ubx_zyy} K Y ``cj Y `h\ Y ``cc__`Ub X `ZYY ``cZ'h\ Y `5dd`Y `K UhW\ "`5g`k Y `a Ybh]cb Y X UVcj Yž`]h``cc_g`U`V]h``]_Y`U`g\ fi b_!Xck b`j Yfg]cb`cZ'h\ Y`cf][]bU`]D\ cb Yž`Ub X`]h g`fY Uggi f]b[`m`fc Vi gh` `UZhY f`U`a cgh`U`mY Uf`k]h\ h\ Y`K UhW\ ž`h\ YfY`bc`g\ UfUhW\ Yg`cb`h\ Y`VcXm`cf`g\ WfYYbž`U`h\ ci [\h\ Y`Vf][\hYf`cZ'ci f`hk c`Gdcfh`6Ub X`ghfUdg`]g`ghUfh]b[`hc``cc_`U`V]h [fi VVm'$

5dd`Y`XcYgb h`fYWca a YbX`Xi b_]b[`mci f`5dd`Y`K UhW\`Z]fgh![Yb`]b k UhYf"`K \]`Y`h\Y`k UhW\`]g`fUhYX`Ug`k UhYf!fYg]ghUbh`hc`h\Y`\DL + ghUbXUfXž'k \]W\`g\ci `X`a YUb`]h`k]```gi fj]j Y`]b`k UhYf'i d`hc`U XYdh\`cZ'%`a YhfY`Zcf`i d`hc`' \$`a]bi hYgž'5dd`Y`XYgWf]VYg`]h`Ug gd`Ug\!`UbX`k UhYf!fYg]ghUbh`Vi h`bch`k UhYfdfccZ "`Gc`]h g`c_`hc i gY`]h`]b`h\Y`g\ck Yf` `Ug`5dd`Y g`79C`H]a `7cc_`UddUfYbh`m`XcYg `Vi h`]h g`bch`hc`VY`hU_Yb`gk]a a]b["

<Uj]b['gU]X'h\Uhž'd`Ybhm'cZ'fYW_`Ygg'fYj]Yk Yfg'\Uj Y'XcbY'YI UWh`m
h\Uhž'UbX'k Y j Y'mYh'hc'\YUf'UbmcbY'Wca d`U]b'h\Uh'h\Y]f'k UhW\
k Ug'XUa U[YX'Vm'h\Y'YI dYf]YbW'''K Y'Xcb h'fYWca a YbX'hU_]b[
h\Y'f]g_ž'UbX'mci 'cVj]ci g`m'k cb h'\Uj Y'U'`Y['hc'ghUbX'cb'k]h\
5dd`Y']Z'gca Yh\]b['XcYg'[c'k fcb['g]bW''h\Ymj Y'VYYb'WUfYZi ``hc
cb`m'WU]a ']h g'k UhYf!fYg]ghUbh''6i h']h'UddYUfg'h\Uh'mYgž'h\Y'5dd`Y</pre>

5dd`Y'K UhWX fYj]Yk. GhfUdg

 $K \ \] `Y'k Y fY'cb'h \ Y'gi V Y Wh'c Z'ghf U dg \check{z}'k \ \] W \ \ cb Y'g \ ci `X'mci 'd] W hc' [c'k] h \ \ 'mci f' V Y U i h] Zi `'5 dd `Y' K U h W 3$

 $\label{thm:cgy-zfca} $$H\YfY\UfY\U'k\]XY\Wc\YWh\Coz'ghfUdg'hc'W\ccgY'Zfca\ z']bWi\ X]b[\ .$$ @]b_'6fUWY\Yhz'Gdcfh'6UbXz'@YUh\Yf'@ccdz'7\Ugg]W6i\ W_\Yz'A\ cXYfb 6i\ W_\Yz'A\]`UbYgY'@ccd'UbX'a\ cfY'fYWYbh`mz'Bm`cb'VUbX'UbX'\UbX'\Yfa\ Yg'ghfUdg''$

H\Y'@YUh\Yf'@ccdž'7`Ugg]W6i W_`Yž'@YUh\Yf'@ccdž'AcXYfb'6i W_`Yž KcjYb'Bm`cb'UbX'Gdcfh'6UbX'cdh]cbg'UfY'cZZYfYX']b'ai`h]d`Y Wc`cif'Wkc]Wyg'k\]`Y'h\Y'A]`UbYgY'@ccd'UbX'@]b_'6fUWY`Yh'UfY cb`m'Uj U]`UV`Y']b'hkc'Wc`cifg"'H\Y'Gdcfh'6UbX'WcaYg']b'&& X]ZZYfYbh'Wc`cifg']bWiX]b['V`UW_ž'k\]hYž'd]b_ž'mY``ckž'V`iYž'[fYmž `]aY'[fYYbž``UjYbXYfž'Ubh]eiY'k\]hYž'ghcbY'UbX'a]Xb][\h'V`iYž ZcfYlUad`Y"

5dd`Y'K UhWX fYj]Yk . GWfYYb

K \]`Y`Xc]b['cif'VYgh'hc'YIhYbX'h\Y'k UhW\g'VUhhYfm'`]ZYž'k Y k UbhYX'hc'ZcfWY!ei]h'gca Y'Uddg'UbX'ZcibX'h\Y'a Yh\cX'cZ'Xc]b[gc'XYYd`m'WcibhYf]bhi]h]j Y"

FYhi fb]b['hc'h\Y'gWfYYbž'h\Y'fYgc`i h]cb'XYdYbXg'cb'h\Y'k UhW\mci 'W\ccgY"'H\Y'fYgc`i h]cb'cZ'h\Y'gWfYYb'cb'h\Y'', a a '5dd`Y K UhW\fk\]W\'a YUgi fYg'%"' &']bW\Yg'X]U[cbU``mŁ']g'&+&3' (\$k\]`Y'h\Y'(&a a 'a cXY``cZZYfg'' %&3' - \$'cb'U'gWfYYb'h\Uh a YUgi fYg'%") ']bW\Yg"'6ch\'a cXY`gž'h\YfYZcfYž'cZZYf'U'd]l Y`XYbq]hm'cZ'' &*'d]l Y`q'dYf']bW\ž'cf'dd]"

±b 'Vch\ 'WUgYgž'h\ Y '5dd`Y 'K UhW\ 'gWfYYb']g'cZZ]W]U``m'fUhYX'fcfdYf\Udg'k Y'g\ci`X'gUm'VfUbXYXŁ'Ug'FYh]bU!ei U`]hmž'UbX'ci fgi V 'Y Wh]j Y 'Y I dYf]Yb W\ 'k]h\ ']h'\Ug'VYYb'[fYUh"' ±h g'g\ Ufd'UbX j]j]X`m'Wc`ci fZi`'UbX'k Y j Y'mYh'hc'bch]W\ 'Ubm'd]I Y``Uh]cb"

 $H\Y'hci\W\gWfYYb\UgdYWhg'k\cf_'hYff]Z]WU``m'hcc.']h\ g'\][\ \mbox{ mode} fYgdcbg]j\ Y\Zci\bX'k\ Y'hYbX'hc'ZU```VUW_'cb'c`X'\UV]hgZ gk\]d\]b\['h\fci\[\ \gWfYYbg'k\ \YbYj\ Yf'dcgg]V`Y'Vm'i\ g]b\['h\Y'hc'\W\gWfYYb'Yj\ Yb']Z'U'gWfc``]b\['cdh]cb']g'Uj\ U]`UV`Y'j\]U'h\Y'8\][\]hU`7fck\ b'''\\\\\'a''\ci\[\ \h'hc'\Y'\Ug'YUgm'hc'ei\]h'Uddg'UbX'[\ `UbWYg'Ug']h']g'cb h\Y']D\cbY'''Vi\h']h']gb h'''Hc'ei\]h'Ub'Udd'mci'\Uj\Y'hc'dfYgg'UbX$

5dd`Y`K UhWX fYj]Yk. I gYf]bhYfZUWY

. .

5dd`Y gʻXYg][bʻYl dYfh]gYʻ]gʻcb`mʻUʻga U```dUfhicZik \Uhia U_Ygʻh\Y 5dd`YʻK UhW\ʻghUbXʻci h''5bch\YfʻZUWhcfʻ]bʻ]hgʻZUj ci fʻ]gʻh\Yʻi gYf]bhYfZUWY'''H\YʻdfcV`Ya ia UbmʻWʻffYbhʻga Ufhk UhW\Ygʻ\Uj Yʻ]gʻh\Uh h\Yʻl =]gʻdUW_YXʻcbhcʻUʻh]bmʻX]gd`UmʻUbXʻmci ibYYXʻhcʻa Ub]di`UhYh\cgYʻh]bmʻj]gi U``Y`Ya Ybhgʻi g]b[imci fʻZ]b[Yfgžʻk \]W\ʻUfY]bYj]hUV`m'V][[Yfʻh\Ubʻh\YʻY`Ya Ybhgʻimci fYʻhfm]b[ihcʻhci W\"

6Y`ck 'k Y`Yl Ua]bY`5dd`Y g`a Yh\cX`Zcf`Wcbhfc``]b['h\Y`i gYf]bhYfZUWYž`UbX`h\Y`gcZhk UfY`mci `WUb`Yl dYWh`hc`gYY`cb`h\Y`5dd`Y K UhWX"

 $5dd^Y'K$ UhWX fYj]Yk . I g]b['h\ Y'5dd^Y'K UhWX gWfYYb

CbY'k Um'hc'i gY'h\Y'K UhW\']g'j]U'h\Y'gWfYYb"'Mci 'WUb'gWfc``
Ufci bX'h\Y'gWfYYbž'hUd'cb']hYa g'hc'gY`YWh'h\Ya 'cf'dfYgg'\UfXYf
hc'[Yh'a cfY'cdh]cbg' 'U_]b'hc'i g]b['f][\h'W]W_'cb'U'a ci gY"
J Uf]ci g'[Yghi fYg'Vf]b['i d'ch\Yf'Y`Ya Ybhg'cZ'h\Y'cdYfUh]b[
gmghYa "': cf'Yl Ua d`Yž'; `UbWYg'UfY'UWWYggYX'Vm'gk]d]b['i d'cb
h\Y'k UhW\'ZUWY"

. .

H\YfY g'U`gc'5dd`Y g': cfWY'Hci W\'hYW\bc`c[m'h\Uh'XYhYfa]bYg\ck'\UfX'mci 'UfY'dfYgg]b['h\Y'K UhW\'UbX'k]``'UWh'UW\cfX]b[`m'H\YfY']g'U'X]ZZYfYbWY'VYhk YYb'U'\UfX'dfYgg'UbX'U'g]a d`Y'hUd"fK Y'X]gW'gg': cfWY'Hci W\']b'a cfY'XYhU]`'VY`ck "Ł

5dd`Y`K UhWX fYj]Yk. GdYYX#dYfZcfa UbWY

CbY'dcgg]V`Y'k YU_bYgg'cZ'h\Y'5dd`Y'K UhW\' 'XYdYbX]b['cb'\ck XYa UbX]b['mci f'ghUbXUfXg'UfY'k \Yb']h'Wca Yg'hc'k YUfUV`Y'hYW\' Wci `X'VY']hg'U``!Ufci bX'gdYYX"'Bi a Yfci g'fYj]Yk Yfg'\Uj Y'Zci bX h\Y']bhYfZUWY'g`i [[]g\']b'i gY'UbX'bch]WYX'U'XY`Um'VYZcfY'WYfhU]b UWh]cbg"

I dXUhY', 'GYdhYa VYf'&\$%*.'\dyg\ci`X'VY'bchYX'h\Uh'h\Y VY`ck 'gYWh]cb']g'VUgYX'cb'h\Y'cf][]bU``k UhW\'UbX'bch'h\Y GYf]Yg'%'cf'&'k\]W\'ZYUhi fY'U') \$'dYfW'bh'ZUghYf dfcWYggcf"

K Y j Y 'U`gc 'Zci bXž'Ug`\Uj Y 'a Ubmi gYfgž'h\Uh'h\]fX!dUfhm'Uddg'WUb gca Yh]a Yg'VY'g`ck 'hc 'ghUfhi d"'k UhW\CG'&'`cc_YX'hc 'gdYYX'i d h\]fX!dUfhm'Uddg'Vm'U``ck]b['h\Ya 'hc'fi b'bUh]j Y`m'cb'h\Y'5dd`Y k UhW\ž'Vi h'Ug'k Y 'a Ybh]cb']b'ci f'k UhW\CG'&'gYWh]cb'cZ'h\Y'fYj]Yk VY`ck ž'k Y'gh]``'Z]bX'Uddg'dfYhhm'g`i [[]g\'UbX'k Y'cZhYb'[]j Y'i d UbX'YbX'i d'i g]b['ci f']D\cbY'Uddg']bghYUX"

5dd`Y`K UhWX`fYj]Yk.`I g]b[`h\Y`8][]hU` 7fck b

5dd`Y gʻgc`i h]cbʻhcʻh\YʻbUj][Uh]cbʻdfcV`Ya ʻ]gʻhcʻi gYʻgca Yh\]b[h\Uh`\UgʻU`k UmgʻVYYbʻUʻZYUhi fYʻcZik UhW\Ygʻ]bʻUʻbYk ik Um"

 $H\Y'X]U``cb'h\Y'g]XY'cZ'h\Y'k UhW\'']hg'dfcdYf'bUa Y']g'h\Y'Wfck b '\Ug'VYYb'Vfci [\h']bhc'h\Y'&%gh'WYbhi fm'UbX'hi fbYX']bhc'k \Uh 5dd`Y'WU``g'h\Y'8][]hU`'7fck b"'H\]g'8][]hU`'7fck b'gc`j Yg'h\Y dfcV`Ya 'cZ'qk]d]b['h\fci [\']Wcbg'cb'U'a]bi qW'`Y'X]qd`Um''$

5dd`Y'K UhWX'fYj]Yk..6UhhYfm``]ZY

± ZUWhž mci fi gY a Umij Ufm" 5dd Y gʻhmd]WU`XUm]bWi XYX U`\U`Z\ci fk cf_ci hž Vi h]Z mci Yl YfW]gY a cfY h\Ub h\Uh mci a Umi gY i dʻh\Y VUhhYfmei]W_Yf `]b 5dd Y gʻhYghgž h\Y VUhhYfm`UghYX *")\ci fgʻXi f]b['Uʻk cf_ci hʻfgc mci 'g\ci `X 'Uh`YUgh'VY 'UV`Y hc fi b h\Uh`a UfUh\cb k]h\ci hʻfi bb]b['ci hˈcZ'VUhhYfm'=Z mci 'i gY h\Y 5dd`Y K UhW\ hc d`Uma i g]Wmci 'k]``U`gc 'Z]bX h\Uh`hc 'VY'U'V]hˈcZ'UVUhYfm`\c["'5dd`Y'[ch'*") '\ci fgʻcZ'Ui X]c 'd`UmVUW_ci hˈcZ'h\Y hYgh'XYj]WY 'VYZcfY']hʻfUb 'ci hˈcZ'dck Yf"

5dd`Y'K UhWX fYj]Yk. 5dd`Y'K UhWX I? df]WY

Df]W]b['j Uf]Yg'XYdYbX]b['cb'h\Y'k UhW\'UbX'ghfUd'mci'W\ccgY": cfa cfY']bZcfa Uh]cb'UVci h'K UhW\'df]WYgž'fYUX'ci f'5dd`Y'K UhW\
Vi m]b['UXj]WY"

 $H\Y'5dd`Y'K\ UhW\&'df]WW'ghUfhg'Uh'v\&) - i]b'h\Y'l?/h\Uhg'Zcf'h\Y', a a '5dd`Y'K\ UhW\&'Gdcfh'k]h\'U'd`Ugh]WVUbXž'UbX'v&- - 'Zcf'h\Y' (&a a 'j Yfg]cb"'H\Y'ghU]b`Ygg'ghYY``5dd`Y'K\ UhW&'ghUfhg'Uh'v(+-UbX'h\Y'bYk\Yf'5dd`Y'K\ UhW&'<math><$ Yfa\ Yg'ghUfhg'Uh'v\%\$\$\$ž'k\]`Y'h\Y' %, !\WUfUh'[c`X'5dd`Y'K\ UhW&'<Xfa\ Yg'ghUfhg'Uh'Ub'YmY!k\ UhYf]b[v, ž\$\$\$"

CIF'J9F8=7H

H\Y'5dd`Y'K UhW\']gb h'h\Y'Z]fgh'Yj Yf'ga Ufhk UhW\ž'UbX']h'XcYgb h fYU``m'Xc'Ubm\]b['f]j U`'dfcXi Whg'Xcb h'Xc"'6i h'k \Uh']h'XcYg'Xcž']h XcYg'Ug'k Y``'Ug'Ubm'ga Ufhk UhW\'ci h'h\YfYž'h\Ub_g'hc'5dd`Y g i gYf']bhYfZUWY'YI dYfh]gY"' \denomina g'U'g`]\w_'XYj]\W'hc'i gYž'U`h\ci [\'mci g\ci `X'VY'k UfbYX'h\Uh']h']gb h'Wca d`YhY`m']bhi]h]j Yž'dUfh]\W'`Uf`m'Uh Z]fgh"'K]h\'i gY']h'k]``'VY\wca Y'a cfY'ZUa]`]Uf'UbX'i gYf!Zf]YbX`m"

8 YhU]`g

6Ygi WXYb G]Y i bg Ui Z. _fmdhc!bYk g"Uh