

K Ufi a '6]hWc]b 'gW\`]a a Yf ']gh U`g Y]b Dcbn]!GW\Ya U ']a 'A UXcZZ!Gh]`

8]Yg']gh'Y]b'; UghVY]hfU['jcb'FcVYfh'A W7Ui `Ymž'b]Wh
UbgÀgg][Yf'GYb]cf: Y``ck 'Ua'; `cVU``8Yj Y`cda Ybh'Dc`]Wh
7YbhYf'XYf'6cghcb'I b]j Yfg]hmi bX'Uggcn]]YfhYg'A]h[`]YX'XYf
: U_i `hÀh'ZØf'; YgW\]W\hY'XYf'I b]j Yfg]hmcZ'CI ZcfX"'=\D'X]YgYa
6Y]hfU['Uf[iaYbh]Yfh'A W7Ui `Ymž'XUgg'XYf'JYf[`Y]W\'jcb
6]hWc]b'a]h'Y]bYa'Dcbn]!GW\YaU'[Y[YbØVYf'Dcbn]!
GW\YaUhU'ibZU]f']gh''6]hWc]b'\Uh'gY]b'5``nY]h\cW\'jcb'*-"\$\$\$
I G8'j ca'-"Bcj YaVYf'&\$&%'ØVYfgW\f]hhYb"'5a''YfghYb
8YnYaVYfkcW\YbYbXY'Yf`]hh'6]hWc]b'Y]bYb'\YZh][Yb':`Ug\!
7fUg\'jcb'%\"\$\$\$'I G8ž'U`g'?cbhYb'ØVYf'[Y\YVY`hY
Dcg]h]cbYb'[YgW\`cggYb'kifXYb"IbX'XcW\ž'gY`Vghinia
U_hiY``Yb'DfY]g'jcb'(-"\$\$\$'I G!8c``Uf'k]fX'Yg'jcb'XYb
; ÀghYb'XYf':]bUbnbUW\f]W\hYb'kY]hYf\]b'U`g'XUg
`Y]ghib[qghÀf_qhY'5gqYh'XYf'`YhnhYb'/Uad\Y'`]d/

8] $Yg']gh'Y]b'; UghVY]hfU['jcb'FcVYfh'AW'Ui`Ymž'b]WXh'UbgÅgg][YfGYb]cf': Y``ck'Ua'; `cVU``8YjY`cdaYbh'Dc`]Wm'7YbhYf'XYf'6cghcblb]jYfg]hm'ibX'Uggcn]]YfhYg'A]h[`]YX'XYf': U_i`hÀh'ZØf'; YgWX]WXhYXYf'Ib]jYfg]hm'cZ'ClZcfX''' <math>\Rightarrow$ X]YgYa'6Y]hfU['Uf[iaYbh]YfhAW'Ui`Ymž'XUgg'XYf'JYf[`Y]WX'jcb'6]hWc]b'a]h'Y]bYa'Dcbn]!GWXYaU'[Y[YbØVYf'Dcbn]!GWXYaUhU'ibZU]f']gh''

8]Yg'k]XYfgdf]Wkh'XYf``Ub['À\f][Yb'g_Ydh]gWkYb'5bg]Wkhij]Y`Yf

2_cbca Yb'i bX'UbXYfYfž'XUgg'6]hWc]b']b'K]f_`]Wk_Y]h'Y]b'Dcbn]!

GWkYa U']gh''8Yf'VfUg]`]Ub]gWkY' +bZcfa Uh]_Yf'*cf[Y'Ghc`Z]']gh'Y]bY

Gh]a a Yž'X]Y'X]Yg'VY\Ui dhYh''GY]bY'5bg]Wkh'VUg]Yfh'Ui Z'XYb

Zc`[YbXYb'6YcVUWkhi b[Yb.'

- %" 5b`Y[Yf'_Ui ZYb']b'9fk Ufhi b['j cb'; Yk]bbYb"
- &" 8]YgY'9fk Ufhi b['k]fX'Xi fW\'X]Y'; Yk]bbY'XYfYf Ui ZfYW\hYf\U`hYbž'X]Y'Ui gnU\`Yb"
- ' " 5VYf'Yg'[]Vh'_Y]bY'YI hYfbY'EiY``Y'ZØf'X]YgY'; Yk]bbY/'g]Y ghUa a Yb'Ui ggW\`]Y»`]W\'Ui g'BYi]bj Ygh]h]cbYb"
- ("IbX:X]Y:6YhfY]VYf:bY\aYb:Y]bYb:[fc»Yb:HY]`:XYg; Y`XYg a]h"

 $5``X]Yg'_`]b[h'k U\f''5VYf'k Ybb'g]Y'6]hWc]b'U`g'Dcbn]!GWXYa UVYnY]WXbYbž'g]bX'?f]h]_Yf'k c\``]b'nk Y]Yf`Y]'<]bg]WXh'ni 'ZfYi bX`]WX''9fghYbg'\Uh'6]hWc]b'b]WXh'XUg'[`Y]WXY'9bXgd]Y``k]Y'Y]b'Dcbn]!GWXYa U''Nk Y]hYbg'ghY``h'Yg'Ui g'Y]bYf'VfY]hYb'[YgY``gWXUZh`]WXYbDYfgdY_h]j Y'Y]b'ni h]YZgh'bY[Uh]j Yg'Gi a a Ybgd]Y``XUf''$

a [fò» hYb i bX k U\fgWXY]b`]WX 'Ua '`Àb[ghYb ``Ui ZYbXYb 'Dcbn]!

Dfc[fUa a 'XYf'; YgWX]WX hY 'nU\`hY '6Yfb]Y 'A UXc ZZ'a cbUh`]WX 'fi bX

Y]b 'DfcnYbh" '9f'Vch'Ubž'XYb 'HY]`bY\a Yfb 'gY]bYg 'GmghYa g'gck c\`

X]Y i fgdfØb[`]WX ']bj Ygh]YfhY 'Gi a a Y 'U`g'Ui WX 'X]Y 'FYbX]hY

XUfUi gʻUi gni nU\`Yb"' ⊕ Zc`[YXYggYbʻ_cbbhYʻXUgʻGmghYa 'Y]bYb Fi bʻYf`Y]XYbʻi bXʻhUhʻYgʻUi W\/8]Yʻ; fc»Yʻ:]bUbn_f]gYʻj cbʻ&\$\$, ZØ\fhYʻni 'Y]bYfʻ? Ug_UXYʻj cbʻF ØW_nU\`i b[YbʻXi fW\'X]Y HY]`bY\a Yfʻi bX'ni a 'Ni gUa a YbVfi W\'XYgʻGmghYa g"

5VYf`X]Y`5i Z`Ògi b[`j cb`A UXcZZg`D`Ub`\Uh`g]W\`Ui Z[fi bX`XYf VYa Yf_Ybgk YfhYb`i bX``Ui ZYbXYb`; Yf]W\hgj YfZU\fYb`ØVYf`gY]bYb Ni gUa a YbVfi W\`\]bUi g`Ui g[YXY\bh"\8]YgY`\UVYb`A UXcZZ`gY`Vgh ØVYf`YVhž`XYf`5bZUb[`&\$&%`ghUfV"

 $\label{eq:linear_control_of_con$

a; Y[YbgUhn'ni '=bj Ygha Ybhg'VY]'A UXcZZ'k]fX'6]hWc]b'b]Wkh'U`g Y]b_ca a YbgghUf_Yf'J Yfa Ò[Ybgk Yfhž'gcbXYfb'U`g'Yk][Yf Bi ``_i dcb'[Y_Ui Zh''A]h'UbXYfYb'K cfhYbž'Yg'j Yfgdf]Wkh'b]Wkhg'U`g `Ui ZYbXY'FYbX]hY'i bX'fY]Zh'b]Y'a]h'Y]bYf'YfZcfXYf`]WkYb HYfa]bU`nU\`i b["'8UfUi g'Zc`[hž'XUgg'Yg'_Y]bYb'Fi b'Yf`Y]XYb_Ubb"'8]Y'Y]bn][Y'A Ò[`]Wk_Y]hž'k]Y'g]Wk'Y]b'6]hWc]b!6Yg]hnYf Ui gnU\`Yb'`UggYb'_Ubbž']gh'XYf'J Yf_Ui Z'Ub'Y]bY'UbXYfY'DYfgcb"

8 Yf'Ni gUa a YbVfi W\'j cb'6]hWc]b'k ØfXY'[Ubn'UbXYfg'Ui ggY\YbU'g'XYf'j cb'Dcbn]'cXYf'A UXcZZ"'9]b'a Ò[`]W\Yf'5i g`ÒgYf'_ÒbbhY'XYf Ni gUa a YbVfi W\'Y]bYg'[fc>Yb'gc[YbUbbhYb'GhUV`YWc]bž'X"'\"9fgUhn!I G!8c``Ufž'X]Y'Ui Z[Y_ca a Yb'g]bXž'i a 'Y]b'6Uf[Y`XVY]b'ZØf?fmdhck À\fi b[ghfUbgU_h]cbYb'VYfY]hni ghY``Yb"'8]YgYibfY[i`]YfhYb'; Y`Xa Uf_hZcbXg'ki fXYb'U`g'8c``Uf!GhY``j YfhfYhYfa]h'g]W\YfYb'JYfa Ò[Ybgk YfhYb'j Yf_Ui Zhž'X]Y']\fYb'Ui gghY\YbXYbJYfv]bXYb

XUgg`Y]b`[fc»Yf`GhUV`YWc]b` XUg`; Y`X`Vf]W\h ž`k]Y`Yg`&\$\$, `VY]
Y]bYa`fY[i`]YfhYb`; Y`Xa Uf_hZcbXg`XYf`: U```k Ufž`XYf`@Y\a Ub!
DUd]YfY`\]Y`h"`8]Yg`_ÒbbhY`X]Y`[YgUa hY`²_c`c[]Y`j cb`?fmdhc`gc
Xi fW\Y]bUbXYf`Vf]b[Ybž`XUgg`Yg`_Y]bY`; YVchY`ZØf`6]hWc]b
UV[YVYb"`8Yf`A Uf_h`_ÒbbhY`Ui Z`i bVYgh]a a hY`NY]h`gW\`]Y»Yb"`

±b·X]YgYa :: U``[ÀVY 'Yg'_Y]bY ``Ub[k]Yf][Yb·fYWkh`]WkYb 6Ya Ø\i b[Ybž'X]Y 'Yb][Yb·ni 'j YfZc`[Ybž'X]Y']\fY '6]hWc]bg ZfØ\nY]h]['Y]b[Y`Ògh'\UVYbž'i a ']\fY'; Yk]bbY 'Ub·X]Y 'Yb][Yb i a ni j YfhY]`Ybž'X]Y 'bcWk '6]hWc]bg 'VYg]hnYb" '6Yg]hnYf'j cb·6]hWc]b \ÀhhYb′_Y]bYb′5bgdfi Wk 'Ui Z'X]Y 'Yb][Ybž'X]Y 'ZfØ\'[Y_Ui Zh'i bX j Yf_Ui Zh'\UVYb"

gY]bYa '7Ug\Zck' A\bY'h'6]hWc]b'Y\Yf'Y]bYa 'DYbbmlGhcW!
Di a d!UbX!8i a d!GWXYa U'U'g'Y]bYa 'Dcbn]!GWXYa U"'# Y]bYa
Di a d!UbX!8i a d!GWXYa U'Yfk YfVYb' < AbX'Yf']a '; fi bXY'k Yfh'cgY
5_h]Ybz'fYXYb'g]Y'\cWX'i bX'\UbXY'b'g]Y'a O[']WXYfk Y]gY
i bhYfY]bUbXYf'ni 'ghY][YbXYb'DfY]gYbz'VYj cf'g]Y'g]Y'Ub'X]Y'Yb][Yb
k Y]hYf[YVYbz'X]Y'j ca '; YgWk Ahn'i bX'XYf'DfY]gVYk Y[i b[
Ub[Ync[Yb'k YfXYb"'K]Y'XUg'Di a d!UbX!8i a d!GWXYa U'YfgWX']Y*h
6]hWc]b'XYb'fY]bYb'K i bgWX'bUWX'?Ud]hU`[Yk]bbYb"'? Ai ZYf
ObbYb'XYb'5bV']W'j cb': fYi bXYbz'X]Y'ØVYf'BUWXh'fY]WX'k YfXYbz'
b]WXh'YfhfU[Yb.'G]Y''Y]XYb'i bhYf'U_i hYf'5b[ghz'Yhk Ug'ni
j YfdUggYb'fi CAC'L'''5i Z'YXYb': U'``a UWXh'6]hWc]b'_Y]b'
J YfgdfYWXi b[Yb'i bX'_Ubb'b]WXh'YbXYbz'k Ybb'Y]b'Dcbn]!GWXYa U
YbXYh'''</pre>

#bink Y]hYfi@]b]YiVYghY\hiY]bik Y]hYfYfi[fc» YfiI bhYfgWk]YX
nk]gWkYbi6]hWc]bii bXiY]bYa iDcbn]!GWkYa UiXUf]bžiXUggiYfghYfYg
Ui giU[[fY[]YfhYficXYfigcn]UiYfiG]WkhY]biGd]Yiia]hibY[Uh]jiYb
GiaaYbi]gh"i#biXYaiAU» Yži]biXYaiYWkhYiFYggcifWYbijiYfVfUiWkh
k YfXYbžiiai6]hWc]biniai@UiZYbiniiVf]b[Ybži]ghiYgigci_cghgd]Yi][ž
k]YiYgiAUXcZZgiNk Y]!icXYfi8fY]!AUbb!6Yhf]YVib]Wkhik Uf"iK Ug
AUXcZZiUi gigcn]UiYfiG]WkhiUi gigY]bYaiDiUbi\YfUi g[YbcaaYbii bX
gWki]Ywi]Wki_cbgiaa]Yfhi\Uhži]ghiY]bYiIajiYfhYjiib[i]biY]bYa
BiiigiaaYbgd]YiifkYfiHfYi\AbXYfijYf_UiZhYigY]biDYbh\cigYŁ"
8]Yij]YfhYi6YcVUWkhib[ijcbiGhciZ]žiXUggiX]Yi6YhfY]VYfiY]bYb
[fcwYbiHY]iiXYgi;YiXYgikY[bY\aYbiiZUgghiAUXcZZgi9]bbU\aYb

i bX`X]Y`9]bbU\a Yb`XYf`6]hWc]b!A]bYf`ni gUa a Ybž`UVYf`X]YgY`g]bX]b`k]fhgW\UZh`]W\Yf`<]bg]W\h`gY\f`i bhYfgW\]YX`]W\"`

K]Y '\ c W\ 'g]b X 'X]Y '? c ghY b 3 '5 b ZUb ['&\$ & %' V Y n] ZZY fhY 'Ghc `Z] 'X]Y
_i a i `]Y fhY b 'N U\ `i b [Y b 'Ub 'X]Y '6]hWc]b!A]b Y f 'gY]h '&\$\$- 'Ui Z'%)
A]``]UfXY b '8 c ``Uf" '6 Y]a 'X Ua U`][Y b '6]hWc]b! DfY]g 'V Y n] ZZY fhY 'Y f 'X]Y
9f\ O\ i b ['X]Y g Y f 'Gi a a Y 'Ui Z'Yhk U'' \$ 'A]``]c b Y b '8 c ``Uf' d f c 'HU[ž k ca]h'\ Ui d hg À W\ `]W\ 'Ghf ca 'V Y n U\ `h'k]f X "'

6Y] XYb \Yi hY \O\YfYb 6]hWc]b!DfY]gYb k AWXgh XUg @ WX gY Yf" I b[YZA\f'-\$\$ bYi Y 6]hWc]bg dfc HU[VYbOh][Yb XYb [fO) hYb HY] j cb () A] Ycb I G!8 c Yuf dfc HU[Ub Ghfca "Gca]h VYhfA[h X]Y bY [Uh]j Y Gi a a Y]a 6]hWc]b!Gd]Y a Y\fYfY nY\b A] Ycli bX ghY][h Ui Z ØVYf Y]bY A] Ycli gV & c Yuf dfc A cb Uh K Ybb XYf 6]hWc]b!DfY]g Ui Z Bi Yini gUa a Yb Vf]WX hž k Øf XYb X]Y; Yk]bbY XYf Yb][Ybž X]Y j Yf_Ui Zh\UVYbži a X]YgY k UWXgYb XY Gi a a Y \]bhYf XYb J Yf i ghYb XYf \b\UVYf ni fØW_V Y]VYb "6]hWc]b a]h Y]bYa Dcbn]!GWXYa U c XYf Y]bYa Di a d!Ub X!8 i a d!GWXYa U ni 'j Yf[Y]WXYb ZYY hY hž XYa ?fmdhck A\fi b[ggmghYa ini 'gWX a Y]WXY b"

 $5 VgWX `]Y * YbX `a i gg `Y]bY `k]fhgWX UZh`]WX Y `5bU`mgY `j cb `6]hWc]b \\ gY]bY `9]bn][Ufh][_Y]h`]b `XYf `; YgWX]WX hY `XYf `A Ub]Yb `UbYf_YbbYb" \\ 5 `g `GdY_i `Uh]cbgcV `Y_h`]gh `6]hWc]b `]b `XYa `A U *Y `VY]gd]Y ``cgž `]b \\ XYa `Yg `Xcfh`b]WX hg `[]Vh" `8]YgY `dcgha cXYfbY `A Ub]Y `V]YhYh`\c\Y \\ DfY]gY `ZØf `9]b hfA[Y `]b `b]Ya UbXYg `HUVY``Yb_U`_i `Uh]cb" `9]b `Yk][Yg \\ Bi ``!; i hgWXY]b `]gh `b]WX h`U`g `GWXYfnž `gcbXYfb `U`g `6]``]cbYb!8c``Uf! \\ 5b`U[Y `YbhghUbXYb" `=a `; Y[YbgUhn `ni `Y]bYa `Dcbn]!GWXYa U`_Ubb \\ 6]hWc]b `b]WX h`]b `Y]bYa `Fi b `YbXYb"`$

6Y] Y]bYa '7fUg\'\UVYb'X]Y' \(\frac{1}{2}\)\UVYf'j cb'6]hWc]b'_c``Y_h]j 'XUg j Yf`cfYbž'k Ug'g]Y'XYb'A]bYfb'ZØf']\fY'6]hWc]bg'VYnU\`h'\UVYb'' 8]YgYf'6YhfU['_Ubb'bUW\'6YfØW_g]W\h][i b['XYf' \(\frac{1}{2}\)ZUh]cb'b]W\h k Y]h'j cb'XYa 'i fgdfØb[`]W\'VY]'A UXcZZ'Ub[Y`Y[hYb'6YhfU[YbhZYfbh'gY]b'''5VYf'6]hWc]b! \(\frac{1}{2}\)\UVYf'k YfXYb'b]Ya UbXYb'\UVYbž i a 'X]YgY'Gi a a Y'ni fØW_ni ZcfXYfb.'G]Y'k]fX'Y]bZUW\']b'FUi W\Ui Z[Y[Ub[Yb'gY]bž'Y]b'gcn]U'Yf'JYf'i gh'''8]Y'6Yg]hnYf'j cb'6]hWc]b k ØfXYb'g]W\'XUbb'bi f'k ØbgW\Ybž'Yg'k ÀfY'Y]b'Dcbn]!GW\Ya U [Yk YgYb'''

EiY``Y.::]bUbW]U``H]a Yg

8 Y hU]`g

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. '_fmd hc!bYk g"Uh