

A]WX UY``Fi V]b // Ua d / G6: g 6U\ Ua Ug! 6fca UbW

9]bY'5ba Yf_i b['ni a '5bZUb[.''8]Yg']gh'X]Y'`YhnhY'88'XYg $$$\line Y = $\line Y = \li

 $K \ Ybb \ G]Y \ [YfbY \ 88 \ YgYb \ ZYfa \ i \ h][Yb \ G]Y \ V]hhY \ AfY \ ?c \ Yf[Yb \ ZO \ YfYb \ BX \ ZYf \ ZYf \ Yfb \ YfYb \ YfYb \ BX \ YhhYfg \ Ubni \ aY \ XYb \ Yf' \ X \ JYf'' \ X \ Jf' \ XUf \ ZYf'' \ ZYf$

8 Ub_Y ZØfg @YgYb i bX gW ObY : Y]YfhU[Y°

: fUb WYgWU: fY]hU[

 $=a \ \ Yi \ h][Yb B Yk g YhhYf.$

A $]WXUY`FiV]b']gh'_Y]b': HL!: UbUh]_Yf'a Y\f$

A]WX UY``FiV]b`]gh`XYf`i`h]a Uh]j Y`BYhnk Yf_Yf"`8Yf`A Ubb`\]bhYf XYa : UbUh]_Yf`8Ug`GdcfhUfh]_Y`]a dYf]i a `\Uh`Y]b`n]Ya`]WXYg XhgYh!@YVYbž`k]Y`]b`X]YgYa `DfcZ]``XYf`BYk `Mcf_`H]a Yg Ui Z[YnY]WXbYh`k]fX"`

Fi V]b 'j YfVfUW\hY '`YhnhYg'\J\f'Ui W\ 'NY]h'a]h'XYa 'VYfØW\h][hghYb 6Yk c\bYf'XYg'\g\gY`ghUUhYgž': HL '; fØbXYf'GUa '6Ub_a Ub!: f]YX" 9]bY'CZZYb`Y[i b['XYg'\gc`j Ybnj Yfk U`hYfg'XYf'[YZU``YbYb'?fmdhc! 6ÒfgY'Ua 'A]hhk cW\UVYbX'Yf[UVž'XUgg': HL 'Y]b'\dj Yghcf']b : UbUh]Wg']ghž'Y]bYa 'I bhYfbY\a Ybž'XUg'Gi ``]j Ub'/7fca k Y```fYdfÀgYbh]Yfh"

88 \ Uhry]b][Y'8YhU]`g'ØVYf'X]Y'6Yn]Y\ib['nk]gWkYb'XYb'VY]XYbYfZU\fYb"'@UihrY]bYf'a]h'XYf'G]hiUh]cb'jYfhfUihYb'DYfgcb'hfUZYbg]Wk'FiV]b'ibX'G6: '9bXY'&\$&%ž'U`g'FiV]b'Y]b': Yf]Yb\Uig'VUihY'5`VUbm'FYgcfhž'X]Y'jcfbY\aY'; YaY]bXYž']b'XYf'g]Wk': HLb]YXYf[Y`UggYb'\UhhY"'

A]WX UY``Fi V]b 'ghY W_hY '%\$ 'A]``]cbYb '8 c``Uf']b ': HL '| '? Yj]b A Uni f#, Yhhm'=a U[Yq'Zcf': UbUh]Wq

Fi V]b 'ghY W_hY '%\$ 'A]``]cbYb '8 c``Uf']b': HL ž'Y]bY 'NU\`ž'X]Y]bnk]gW\Yb'Ui g[Y`ÒgW\h'k i fXY"'@Ui h'X]YgYf'DYfgcb'\Uh': HL YXcW\']a '; Y[Ybni ['%\$ 'A]``]cbYb'8 c``Uf']b': UbUh]Wg'[YghYW_h" @YhnhYfY ' \Rightarrow j Ygh]h]cb'\Uh'g]W\'j]Y``VYggYf'Ybhk]W_Y`h"

8]Y'%\$'A]``]cbYb'| G!8c``Uf'jcb': HL'kifXYb']b'Y]bY A]hhY`VYqW\UZZib['5bZUb['&\$&&']bjYgh]Yfhž'X]Y': UbUh]Wg'a]h'&+ A]``]UfXYb`I G!8c``Uf`VYk YfhYhY"'Jcf'bi f'k Yb][Yb`K cW\Yb`gh]Y[X]YgY`6Yk Yfhi b[']b`Y]bYf'k Y]hYfYb`Fi bXY'Ui Z'' %'A]``]UfXYb`I G!8c``Ufž'Ub`XYf'Ui W\gc`W\Y'VYhY]`][h'k UfYb`7`YUf`U_Y!

< Ui dhghUXhž'@Òk YbVUi a ž'GcZh6Ub_'i bX'G]`j Yf'@U_Y!

DUfhbYf"

8]Y: UbUh]Wg! *bj Ygh]h]cb 'XYf'? fmd hc[fi ddY']gh'bi b 'HY]` XYf': HL!?cb_i fga UggYž'X]Y 'XYf'bYi Y'7\YZ'XYg'l bhYfbY\a Ybg'\Uh' \\ \Lambda \text{Ubb'FUm'} == Gi ``]j Ub'/ '7fca k Y``i bX'6Ub_Yf'VY]'DYfY``U \\ K Y]bVYf['DUfhbYf'; Y`X'j YfX]YbYb'k c``Ybž'i a '?cbhc]b\UVYf Ui gni nU\`Yb"'

: $UbUh]Wg^Y\bhY^Yg^UV\check{z}g]WX^ni^gY]bYf^6Yn]Y\ib[^ni^: HL^ni^Ai^Yfb'']$

 $\label{thm:continuous} Fi \ V]b \ j \ YfVf]b \ [h'j]Y``Y]WA h a \ Y \ f'NY]h'Ui \ Z'XYb'6U \ Ua \ Ug \ UYf'G6: b]WA h''B UWA XYa 'Yf'Ua '8 cbbYfghU[']b 'B YK 'Mcf_'Ub[Y_ca a Yb ib X']b'6i b XYg \ UZh'[Ybca a Yb'k cfXYb'k Uf \ Z'Y[hY'Y]b'F]WA hYf Y]b Y'? Ui h]cb']b'< O\Y'j cb'&) $ A]``]cbYb'8 c``Uf'ZØf']\b'ZYgh \ X]Y gY]bY'5bk YgYb\Y]h']a '< Ui g'gY]bYf'9`hYfb']b'DU`c'5`hc \ Z'U`]Zcfb]Yb \ Z'YfZcfXYfh'''$

Nk Y] "gY] bYf 'Yb[ghYb 'A]hUfVY]hYfž '7 Ufc`]bY '9``]gcb 'i bX'; Ufm K Ub[ž'k]fX "Ui W\ 'b]W\ h "gc 'gW\ bY`` 'ni fØW__Y\fYb" '8 Ug 'DUUf'\ Uh g]W\ 'XYf '6Yhfi [gj cfk ØfZY 'gW\ i `X]['VY_Ubbh'i bX 'g]W\ 'VYfY]h Yf_`Àfhža]h "XYb'l G!6Y\ÒfXYb'ni gUa a Ybni UfVY]hYb"

K]Y 5X]XUg XYb 6U``ZU``Yb `]Y»

8 UgʻzJ\fʻ]ghʻ&\$%* "'? Ub mYʻK Ygh gʻ5`Vi a 'JYfV`UggYbʻXca]b]Yfh X]Yʻ5]fk Uj Ygʻi bXʻ8 fcdgʻXYfʻMYYnm!GbYU_Yf!?c``UVcfUh]cbʻXYg F UddYfgžʻXYfʻni a '8 Yg][bYfʻk i fXYʻ5 X]XUgʻj Yf_Ui ZYbʻg]WXʻgc gWXbY``žʻXUggʻ< ÀbX`YfʻgWX ÀhnYbžʻXUggʻbi fʻ\$ž&) 'DfcnYbhʻXYf \cZZbi b[qj c``Ybʻ?i bXYbʻY]bʻDUUfʻ]bʻX]Yʻ< ÀbXYʻVY_ca a Yb"

]bgd]f]Yfh`\Uh"`8Ug`j YfUb`UgghY`5X]XUg`XUni ž'X]Y`6Yn]Y\i b[Yb`nigY]bYa `dfcZ]hUVY`ghYb'; YgW\ÀZhgdUfhbYf`UVni VfYW\Yb"

8 Ugʻ9bXYʻXYfʻDUfhbYfgW\UZhʻ\UhʻXUni 'VY][YhfU[YbžʻXUggʻXYf 5_h]Yb_i fgʻj cbʻ5X]XUgʻ]a ʻj Yf[Ub[YbYbʻ*J\fʻi a ʻ) ('DfcnYbh [YZU``Ybʻ]gh"5VYfʻXYfʻ9]bnY`\ÀbX`Yfʻ\UhhYʻgW\cbʻ`Ub[Yʻj cfʻgY]bYf Øb[ghYbʻ?f]gYʻa]hʻh]YZg]hnYbXYbʻDfcV`YaYbʻ]bbYf\U`VʻgY]bYf Cf[Ub]gUh]cbʻni '_ÀadZYbžʻVYf]W\hYbʻ9`YUbcfʻC`Wchhʻi bXʻC`UZ GhcfVYW_j cbʻXYfʻ: H"

±b i ±b hY fj]Y k g a]h '%+ 'U_hi Y ``Y b i b X 'Y \ Y a U `][Y b : Ø\ fi b [g_fÀ ZhY b ž j c b 'X Y b Y b 'j]Y `Y 'X U g 'I b hY f b Y \ a Y b 'j Y f `UggY b `\ UV Y b ž 'g U [hY 'X Y f g W X Y]X Y b X Y '7 \ Y Z '? Ugd Y f 'F ÒfghY X 'i b X 'g Y]b 'J c f g hU b X `\ UhhY 5 X]X U g 'g W `Y W h 'U i Z [Y g hY ``hž i a 'X Y b 'K Y g hg hi fa 'ni 'Ø V Y f g hY \ Y b ž G W \ `ØggY `d Y f g c b U ``ni 'Y b h `UggY b i b X 'g] W 'ni 'g Y \ f `U i Z 'X] Y Ni g U a a Y b U f V Y]h 'a]h 'M Y Y n m 'ni 'j Y f `UggY b " `

G]Y'VY\Ui dhYhYb'Ui W\ž'XUg' A UbU[Ya Ybh'Ui g'5b[gh 'XYg gW\Y]XYbXYb'7\YZg'\UVY'X]Y'A]hUfVY]hYf'hfUi a Uh]g]Yfh'i bX'ni Y]bYa '9| cXi g'j cb'HU`YbhYb'[YZØ\fh"

F ÔfghY Xg B UW Zc `[Yfž ``Ub ['À\f][Yf 'Di a U '7 \ Y Z '6 'Òfb; i `X Y b k i f X Y 'b i b 'X Ua]h 'V Y Ui Zhf U[hž 'X]Y 'k UW gY b X Y '? `i Zh nk]g W Y b '5 X]X Ug 'i b X 'g Y]b Y b 'F]j U `Y b 'n i 'V Y _ Àa d Z Y b "

8 Ug'8YVU_Y``\Uh'X]Y'F]g]_Yb'Y]bYf'ni '[fc»Yb'5V\Àb[][_Y]h'j cb Y]bYa 'Y]bn][Yb'DUfhbYf'XYi h']WX'[Ya UWXh"'5`g'GhUXh 5bU`mh]_Yf'H\ca Ug'7\Ui jYh'Zcfa i`]YfhY'Yg'`YhnhYb'A cbUh K U\fgWXY]b`]WX']gh'Y]b'_ca d`YhhYf'FYgYh'XYf'5X]XUg!A Uf_Y YfZcfXYf`]WX"

$\times V'VYkY[h'g]W$

8 Ub '6Yf_cj]hn; YbYfU`'7ci bgY`'XYf'I G5'K Yfhd Ud]Yf!
i bX'6ÒfgYb_ca a]gg]cb9f'k]fX'X]Y'5[Ybhi f'9bXY
 **Jbi Uf'j Yf`UggYb"'A Y[Ub '6UfVYfcXYfnY]h
 ghY``j YfhfYhYbXYf'<Ui dhfYWXhgUbk U`h'XYf'G97ž'k]fX
 gY]bYb'D`Uhn'Y]bbY\a Yb"

- GhÅd\Ub]Y'6fi WYXYf':]bUbnj cfghUbX'j cb'5VfXb@Ui h
 6`cca VYf['d`Ubh'Yfž'XYb'JYfa Ò[Ybgj Yfk U`hYf']b'XYb
 _ca a YbXYb'A cbUhYb'bUWX'Y]bYf'ZØf'XYb'?cbnYfb
 \YfUi gZcfXYfbXYb'NY]h'ni 'j Yf`UggYb"
- HYg`U`\Uh'gY]bYb'7\]bU!7\YZ'[YgWX]W_h'Hca 'N\i ']b'X]Y I G5ž'i a 'VY]'XYf'6Yk À`h][i b['dfcXi _h]cbghYWXb]gWXYf <YfUi gZcfXYfi b[Yb']b'XYf'FY[]cb'ni '\Y`ZYbž'k Ug GdY_i `Uh]cbYb'Ub\Y]nhž'XUgg'Yf'Ui Z'Y]bY'[fÒ»YfY'Fc``Y'U`g 79C'XYg'9`Y_hfcUi hc\YfghY``Yfg'j cfVYfY]hYh'k]fX'9`cb A i g_'k i fXY'`Ui h'FYi hYfg'a]h'Hk]hhYf'UV[Y`Yb_h"

=b hY ``][Y b h ``] Y gh

8]Y '9F!9`]hY '9]b 'gdYn]Y ``Yf'FUi a ']b 'XYf'[YgW\ÀZh][YbBchUi ZbU\a Y 'XYf'BYk 'Mcf_'I b]j Yfg]hm']b 'A Ub\UhhUb']gh'nk Y]
5fhYb 'j cb 'DUh]YbhYb '[Yk]Xa Yh 'XYbYbž'XYfYb '@YVYb'Ui Z'XYa
Gd]Y `'ghY\hž'i bX'J \Dg"'8]Y 'BYk 'Mcf_'H]a Yg'[Y\h'J cfk ØfZYb'bUW\ž
XUgg'XUg'?fUb_Yb\Ui g'GdYbXYfž'HfYi \AbXYf'i bX']\fY': Ua]`]Yb
VYj cfni [h'VY\UbXY`h"

±bX]Yb`cXYf`6ØghY`K À\fYbX`g]W\`7\]bUg`Y]bgh`VfUbXU_hi Y``Yf J Yfa Ò[Ybga Uf_h`Ub[Yg]W\hg`[Ycdc`]h]gW\Yf`GdUbbi b[Yb UV_Ø\`hž`\Uh`Y]b`bYi Yf`; c`XfUi gW\`XYb`[`cVU`Yb`5bghi fa df]j UhYf`6Ub_Yb`Ui g[Y`Òghž'6YfUhYf`ZØf`±bX]Ybg`FY]W\Y Y]bni ghY``Ybž'VYf]W\hYh`6`cca VYf["

@Yi hY'j cb'=bhYfYggY'Jcb'GUa '6Ub_a Ub!: f]YX'V]g'\]b'ni f ?fmdhc!6YhfØ[Yf]b'a]h'XYa 'FUd!BUaYb' F5NN@9?5<B 'V']W_Yb i bgYfY'?c``Y[Yb'j cb'5`d\Uj]``Y'Ui Z'Y]b'*J\f'ZUgn]b]YfYbXYf 7\UfU_hYfY'ni fØW_"

I bX'Y]b: H!:]`a . '8]Y'ŸfU'XYg'`Y]W\hYb'; Y`XYg']gh'j cfVY]": H! £i fbU`]ghYb'gW\`ØggY`b'X]Y'bÀW\ghYb'[fc»Yb'6YXfc\i b[Yb'ZØf XUg'[`cVU`Y':]bUbngmghYa 'Ui Z''

BUW f]W hYbni gUa a YbZUggi b[

; i [[Yb\Y]a 'DUfhbYfg':]bUbn]Yf'GWchh'A]bYfX'gh]fVh']a '5`hYf'j cb *' 'fl HŁ'

9\Ya U`][Yf'7fÅX]h'5[f]Wc`Y!<AbX`Yf'VY\Ui dhYh'fUgg]gh]gW\Y Jcfi fhY]`Y'VY]'XYf'6Ub_'fb`cca VYf[Ł'

8 Y`c]hhY'j cb'Vf]h]gW\Yf'5i Zg]W\hgVY\OfXY'k Y[Yb'G \Rightarrow ! DfØZ b[gUfVY]h'fl HŁ'a]h'; Y`XghfUZY'VY`Y[h'

?fYX]h\UbXY`gj Yf`i gh'j cb'DA cf[Ub'\]b['j cb']bhYfbYb6Yk Yfhi b[Yb'UV'fb`cca VYf[Ł'

 $Hcg\] VUg \cdot VYj \ cfni \ [\ hYf'6]YhYf' VYg]Y[\ Y`h'\%$z*!A]``]UfXYb!8c``Uf! \\ 8 \ Uf`Y\Ybg[\ YgWX AZh'fF Yi \ hYfg#Vca]i \ f]Ł'$

 $\label{eq:helmonton} $H]_{Hc_{-}[]Vh'ni\ \check{z}':\ H!\ \&i\ fbU`]ghYb`VY]'XYf'I\ bhYfgi\ Wi\ b[\ 'j\ cb'@YW_g$ $j\ YfZc`[\ h'ni\ `\ UVYb'fl\ HŁ'$

DcfgWXY'i bX'XYf'B]YXYf[Ub['XYf'XYi hgWXYb'6ÒfgY'fl H'Cd]b]cbŁ'

EiY``Y.:]bUbWJU``H]a Yg

8 Y hU]`g

6Ygi WXYb G]Y i bg Ui Z _fmdhc!bYk g"Uh