

@ilYfYXil.BYk Mcf_g'<][\!9bX! 9][Yb\Y]a _AiZYf'_Y\fYb'nifØW_

I a 'XYb' < UbXY``ni 'hÀh][Ybž'XYf']a 'A U]'&\$&%'[YgWX`cggYb'k i fXYž k UbXhY'g]WX'?Yc[\'XYf'?fmdhck À\fi b['ni ž'a]h'XYf'Yf'Yhk U'%, A cbUhY'ni j cf'VY[cbbYb'\UhhYž'ni '\UbXY`b"'BUWXXYa 'Yf'X]Y?Ud]hU`YfhfU[gghYi Yf'[YnU\`h'\UhhYž'j Yfk YbXYhY'Yf'gY]bY9h\YfYi a !; Yk]bbY'ZØf') \$`DfcnYbh'XYf'5bnU\`i b['XYg'bYi Yb< Ui gYgž'k Ug' < i bXYfhhUi gYbXYb'j cb'8c``Uf'Ybhgdf]WXh''

8 Ua]h']W\ 'Y]bY'5bnU\`i b['`Y]ghYb'_Ubbž'k À\fYbX'a Y]b'[UbnYg; Y`X']b'a Y]bYa 'UbXYfYb' < Ui g']ghž'a i gghY']W\ '[Y\Yb' \chi h\Yf\Q CfhY ž'gU[h'Yf"'5`gc'k UbXhY'Yf'g]W\ 'gY]bYf'X][]hU`Yb'; Y`XV\OfgYni "'

 $\label{thm:condition} I bX ??Yc[\ \]gh'b]WX h'U``Y]b"`A Y\fYfY`A U_`YfZ]fa Ybž'a]h'XYbYb`X]Y : H'gdfUWX ž'VYf]WX hYhYb'j cb'Y]bYf'Ni bU\a Y'j cb'?Ài ZYfbž'X]Y'j cb]\fYb'?fmdhck À\fi b[g[Yk]bbYb'dfcZ]h]YfYb'k c``Yb"'$

K À\fYbX'XYf'DUbXYa]Y'Yfn]Y`hY'?fmdhck À\fi b[g[Yk]bbY'k YfXYb ni bY\a YbX'ni a '?Ui Z'j cb'@ l i g]a a cV]`]Yb']b'A Ub\UhhUb'i bX @cb['=g`UbX'f]a '6]`XŁ'j Yfk YbXYh'| '; Uj]b'NY][`Yf#5`Ua m

O@Ugh:mYUfQK]figU\YbiX]Yi9bhghY\ib[ijcbiOn\YQgidYffY]WXYf?fmdhc!?ÀiZYf žigU[hi; UffYhhi8YfXYf]Ubži8]fYWhcficZiAUf_Yh

±hY``][YbWYiVY]i6fc_YfU[YiGYf\Ubh"i9fibYbbh; YVÀiXYik]YiX]Y
('&iDUf_i5jYbiYi]biAUb\UhhUbgigc[YbUbbhYfi6]``]cbU]fYgiFck
U`gi6Y]gd]Y``ZØfiY]bi; YVÀiXYžiXUgiVY]i?fmdhc!?ÀiZYfbiVY`]YVhi]gh"i

G\Ui b'Cg\Yfz"; YgW\ÀZhgZØ\fYf'XYg'A U_`Yfi bhYfbY\a Ybg'7cfYz
n]h]Yfh'FcgY'<]``z'Y]bY'bYi Y'9bhk]W_`i b[']a 'BcA UX!J]YfhY`'j cb
A Ub\UhhUbz'U`g'VY`]YVh'VY]'?Ài ZYfbz'X]Y'Y]b'DU_Yh'a]h'?fmdhc!

#bj Ygh]h]cbYb'[YhÀh][hi bX'a UbW\a U`'9]b\Y]hYb'ØVYf'&\$'A]``]cbYb
8c``Uf'[Y_Ui Zh'\UVYb"' G]Y'k c``Yb'gYI mz'\c\Y'8YW_Ybz'
i b[`Ui V`]W\Y'5i gV`]W_Y'OUbXQhc``Y'5bbY\a `]W_Y]hYb z'gU[h'Yf"
A UbW\Y'`YVhYb'j cf'nk Y]z'XfY]'\J\fYb'bcW\']b'Y]bYf
9]bn]a a Yfk c\bi b['i bX'gi W\Yb'bi b'bUW\'K c\bi b[Yb'a]h
K ck!9ZZY_h "'

G]Y'k c``Yb'Yhk Ugž'XUg'b]Ya UbX'gcbgh'\UVYb'_Ubb"'G]Y'k c``Yb Yhk Ug'; fc» Yg' 'g]Y'k c``Yb'Y]b'DYbh\ci gY ž'gU[h'A U_`Yf'FmUb GYf\Ubhž'XYf'VY\Ui dhYhž'j cb'Yhk U'Y]bYa '8i hnYbX ?fmdhck À\fi b[g]bj YghcfYb'Ub[YgdfcW\Yb'k cfXYb'ni 'gY]bž'X]Y g]W\'b]W\h'ZØf'hfUX]h]cbY``Y'?fYX]hY'ei U`]Z]n]YfYb'k ØfXYb" A U_`Yf'HU``5`YI UbXYf'gU[hž'Ui W. 'Yf'gY]'j cb'? Ài ZYfb'_cbhU_h]Yfh k cfXYbž'X]Y']\fY'? fmdhck À\fi b[g[Yk]bbY'ZØf' < Ài gYf']b'A]Ua] Ui g[YVYb'k c``hYb'''K À\fYbX'X]Y'a Y]ghYb' \pm bj YghcfYb']\fY ? fmdhck À\fi b['ni Yfgh']b'6Uf[Y`X'i a k UbXY`b'fY]bY'HfUbgU_h]cbž X]Y'XYf'? Ud]hU`YfhfU[gghYi Yf'i bhYf`]Y[h½'i bX'XUbb'hfUX]h]cbY`` ghfi _hi f]YfhY'? Ài ZY'hÀh][Ybž']gh'A]Ua]' `XYggYb'B65!HYUa ']b Y]bYf'5fYbU'gd]Y'hž'X]Y'bUWX'XYf'? fmdhck À\fi b[g! HUi gWXd'UhhZcfa ': HL 'VYbUbbh']gh' `X]Y'<Y]a Uh'j cb @i I i gYbhk]W_`i b[Ybž'X]Y'? fmdhck À\fi b['X]fY_h'U_nYdh]YfYb'''9]b DYbh\ci gY'VY]'5fhYž'Y]bYa '; YVÀi XY']b'Gi fZg]XYž'j Yf_Ui ZhY']a j Yf[Ub[YbYb'A U]'gY]b'i bhYfYg'DYbh\ci gY'ZØf'&&ž) 'A]``]cbYb 8 c``Uf']b'Y]bYa 'fY]bYb'? fmdhck À\fi b[g[YgWXÀZhž'i bX'X]Y 9bhk]W_`Yf'gU[Ybž'XUgg'g]Y'Yg'ZØf'ni _ØbZh][Y'Dfc'Y_hY'U_nYdh]YfYb k YfXYb''

±0\UVYf'j cb'?fmdhck À\fi b[Yb'_ÒbbhYb'YXcW\'VY[]bbYbžX]Y
FYbhUV]`]hÀh']\fYf'DcfhZc`]cg']b': fU[Y'ni 'ghY``Yb' 'i bX'XUg
DchYbn]U`ž'g]Y']b'\UfhY'JYfa Ò[Ybgk YfhY'i a ni k UbXY`b.'8Yf'DfY]g
j cb'6]hWc]b']gh']a 'j Yf[Ub[YbYb'₺J\f'i a 'Yhk U'' \$'DfcnYbh
[YZU``Ybž'9h\YfYi a 'YVYbZU``g"

 $?Yc[\ \]b' = Ui g'[YbUi 'ni a 'f]WXh][Yb'NY]hdi b_h'j Yf_Ui Zh \ \ UVYb' 'i bX'nk Uf'Ub'Y]bYb'5bk U`h'ZØf'X][]hU`Y'J Yfa Ò[Ybgk YfhY"]$

CVk c\`X]Y`NU\`XYf`? Ài ZYf'j cb`? fmdhck À\fi b[Yb']a a Yf'bcW\
gY\f'[Yf]b[']ghž]ghX]Yg'Y]bY'a Yf_k ØfX][Y'9bhk]W_`i b['ZØf'XYb
@ilig]a a cV]`]Yba Uf_h']a 'F Ui a 'BYk 'Mcf_'7]hmž'XYggYb
J Yfa Ò[Yb'g]W\'gY]h'XYf'DUbXYa]Y'j YfÀbXYfh'\Uh''J cf'nk Y]'\\J\fYbk Uf'X]Y'6fUbW\Y']b'Y]bYf': `Ui hY''BYi 'Ybhk]W_Y`hY'K c\bi b[YbžcZh']a 'K Yfh'j cb'a Y\fYfYb'nY\b'A]``]cbYb'8c``Ufž'gW\a UW\hYhYb Ui Z'XYa 'A Uf_h'i bX'V']YVYb'a UbW\a U`''U\fY`Ub['bUW\'XYf': Yfh][ghY``i b['i bj Yf Ui Zh''

5VYf']b'XYb'`YhnhYb'AcbUhYbž'U`g'g]W\'X]Y'GhUXh'jcb'XYa'GW\`U[Yf\c`h'\Uhž'XYf']\f'XifW\'X]Y'7cj]X!%-!DUbXYa]Y'ni[YZØ[h'kifXYž]gh'X]Y'BUW\ZfU[Y'YId`cX]Yfh'']bgVYgcbXYfY'bUW\@lig]aacV]`]Yb']b'AUb\UhhUb'ibX'6fcc_`mb"'

a '*Jbi Uf'k i fXYb'*%\$&'bYi Y'J YfhfÀ[Y'ZØf'A Ub\UhhUbg @ I i g]a a cV]']Yb'i bhYfnY]W\bYh' 'X]Y Yb][Yb'a]h'Y]bYa 'DfY]g'j cb ('A]'`]cbYb'I G!8c'`Uf'cXYf'a Y\f' 'XcddY`h'gc'j]Y`Y'k]Y']a '*Jbi Uf &\$&\$"'@Ui h'Y]bYa '6Yf]W\h'XYg'A U_`Yfg'8ci [`Ug'9``]a Ub'k Uf'XYf 6YhfU[ž'XYf']a 'j Yf[Ub[YbYb'*J\f'ZØf'X]Y'@ I i g]a a cV]`]Yb'XYf GhUXh'Ui g[Y[YVYb'k i fXYž'XYf'\ÒW\ghY'A\f`]W\Y'; YgUa hgi a a Y ZØf'a Y\f'U`g'Y]b'*J\fnY\bh' 'i bX'ZUgh'XcddY`h'gc'j]Y`'k]Y']a '*J\f &\$&\$"'

@Ui h'Y]bYf'Ghi X]Y'j cb'GYf\Ubh'k i fXYb']a 'j Yf[Ub[YbYb'*\] fa Y\f'< Ài gYf'ZØf'ØVYf'%\$'A]``]cbYb'l G!8c``Uf'j Yf_Ui Zh'U`g']b'XYb \\J\fYb'&\\$%- i bX'&\\$&\\$'ni gUa a Yb''' \\\\W\'\UVY'bcW\'b]Y'Y]bYb gc`W\Yb'A Ub[Y`'Ub'E i U`]h\Ahg]bj YbhUf']b'XYb'< Ui dha \\Af_hYb [YgY\Yb \\Z'gU[h'GYf\Ubh!A U_`Yf'7\UgY'@UbXck "'8 Ug'5b[YVch'gY] gc'_bUdd\Z'gU[h'Yf\Z'XUgg'gY]bY'?i bXYb'VY]'5bbY\a`]W_Y]hYb'k]Y df]j UhYa '5i \\YbVYfY]W\'i bX'DZ\OfhbYf'5Vghf]W\Y'a UW\Yb'''

8 Ug'[Y\cVYbY'J]YfhY`'6fcc_`mb'<Y][\hg.'8]Y'FY]W\Yb'g]bX Ui Z[fi bX'\Ò\YfYf'9fgdUfb]ggY'i bX'FYbX]hYb'Ui g'±j Ygh]h]cbYb fY]W\Yf'[Yk cfXYb'| 'A]W\UY`'6fcc_g#5`Ua m

B Ub Whi K i \check{z}^2 _cbca]b VY]a = a cV]`]YbdcfhU``GhfYYh9Ugm \check{z} ZØ\fh X]Y J Yf Øb[i b[XYg @ I i ga Uf_hYg Y]b ZUW\ `Ui Z X]Y `A YbgW\ Yb ni fØW_ `Qa dfcj]b[QZ]bUbn]Y``Y `G]hi Uh]cb `k À\fYbX`XYf`DUbXYa]Y "J]Y`Y `k c\`\UVYbXY`BYk `Mcf_Yf`g]bX`]b `XYb ``YhnhYb `nk Y]`\$J\fYb Ui Z[fi bX`\ \check{O} \YfYf`9fgdUfb]ggY `i bX`FYbX]hYb `Ui g K UW\ghi a gU_h]Yb `fY]W\Yf`[Yk cfXYb `i bX`k UfYb `XUfUi Z j cfVYfY]hYhz'j cb `Y]bYa 'j Yf\U`hYbYb `A Uf_h`ni 'dfcZ]h]YfYb"

5 VYf`XUgʻgY]'b]W\h'ØVYfU```gcž`gU[h'Ki" K]f`gY\Ybž'XUgg`X]Y
DfY]gY`Ua ii bhYfYb`9bXY`XYg`A Uf_hYg`ZU``Yb ž'XYbb`Yg`gY]Yb`X]Y
< Ui gVYg]hnYfž'X]Y` Z]bUbn]Y```Ua 'ghÀf_ghYb'j cb`XYf`DUbXYa]Y
VYhfcZZYb 'gY]Yb"'G]Y`\UVYb'_Y]bY`X][]hU`Yb'; Y`XVÒfgYbž'Ui Z'X]Y
q]Y`ni fØW_[fY]ZYb'_ÒbbYb"

G\]j cb 8i bWUbg 9`hYfbžX]Y j cf Y]bYa `\U`VYb \J\f\i bXYfh'Ui g ; fYbUXU Ui g[Yk UbXYfh'k UfYbž\UVYb]\f\i]\f\X]Y '6YXYi hi b['XYg 9][Yb\Y]a g 'VY][YVfUW\hž'i bX 'gY]h']\fYf' \(\forall \) [YbX'k Uf 'Yg']\f'HfUi a ž Y]bY '9][Ybhi a gk c\bi b['ni 'VYg]hnYb" 'G]Y 'k c``hY 'Y]b 'Ni \Ui gY ']b Y]bYa '; YVÀi XY 'a]h'5bbY\a `]W_Y]hYbž'UVYf'a]h'ghY][YbXYb DfY]gYb" '9g']gh'k]Yž'k Ug'a UW\gh'Xi 3 'G]Y 'ZfU[h''

a 'J]YfhY`A]``6Ug]b']b'6fcc_`mb'ZUbX'g]Y'[YbUi 'XUgž'k cbUW\'g]Y gi W\hY"8i bWUbž'XYf']a 'OZZYbh`]W\Yb'?fUb_Yb\Ui ggmghYa 'XYf GhUXh'UfVY]hYhž'_Ui ZhY']a 'Bcj Ya VYf'Y]bY'9]b!N]a a Yf! 9][Ybhi a gk c\bi b['a]h'; UfU[Y'i bX'5i ZZU\fh'ZØf'&, \$"\$\$\$'8c``Uf" G]Y'gW\Ui h'g]W\'Yhnh'i a 'i bX'g]Y\h''\ b[Y'A YbgW\Ybž'X]Y 8cddY'j YfX]YbYf!<Ui g\U'hY'VfUi W\Ybž'i a 'VYei Ya 'ni '`YVYb' 'g]Y _ObbYb'XUg'b]W\h'U``Y]bY"'I bX'g]Y'VfUi W\hY'Ui W\'<]`ZY"

8 i bWUb 'VYk UfV'g]W\ 'ZØf'XUg' < ca Y:]fgh! 5 bnU\`i b[g\]`ZYdfc[fUa a 'XYg'BYk 'Mcf_'7]hm'8YdUfha Ybh'cZ < ci g]b['DfYgYfj Uh]cb'UbX'8Yj Y`cda Ybhž'XUg'V]g'ni '%\$\$"\$\$\$'I G! 8 c``Uf'ZØf'X]Y'5 bnU\`i b['cXYf'X]Y'5VgW\`i gg_cghYb'ZØf ei U`]Z]n]YfhY'? Ài ZYf'VYfY]hghY``h"' $C \bY'X]YgY' <]`ZY'k \AfY'Yg$ a Y]bYf'A Y]bi b['bUW\'b]W\ h'a $\bO()$ `]W\'[Yk YgYb \bZ'' gU[h'g]Y"

 $B\ Yk\ Mcf_7\ hm\ Uh\ Y\ bY\ VY_UbbhY\ 9fgWk\ b[\]b[\]Wk_Y\ hg_f]gY\ z\ UV\ Yf\ X\ Y\ DUbXYa\ Y\ Uh\ XYa\ K\ c\ bY\][\ Ybhi\ a\ k\ Y\]hY\ fY\ <\]b\ XYfb\ ggY\]b\ XYb\ K\ Y[\ [\ Y\ Y\]hgd\ Uhn\ j\ Yf\ cfYb\ z\ a\ i\ gghYb\ Ui\ g\ XYf\ GhUXh\ k\ Y[\ n]Y\ Yb\ cXYf\]\ fY\ <\ Ai\ gYf\ nk\ Ub\ [\ gj\ Yfd\ ZAb\ XYb\ z\ gU[\ h\ K\ i\ "\ 9]b\ [\ fO\ YfYg\ 5b\ [\ YV\ ch\]b\ X\]YgYf\ A\ Uf_hghi\ ZY\ ZO\ fh\ XUbb\ ni\ b\]YXf][\ YfYb\ DfY\]gYb\ z\ K\ Ybb\ X\]Y\ B\ UW\ ZfU[\ Y\ b\]Wk\ h\ gc\ [\ fc\ >\]gh"$

 \pm 0'; Y[YbXYb'k]Y'EiYYbg'\Uh'X]Y'DUbXYa]Y'XYa'K c\bY][Ybhi a k Y]hYfY'<]bXYfb]gqY']b'XYb'K Y['[Y`Y[h']'G\i hhYfghcW_'#fV`Za f

EiYYbg'kifXY'jcb'X]YgYf'FYU`]hÀh'VYgcbXYfg'\Ufh'[YhfcZZYb'']bXYb''%&'AcbUhYb'V]g'\JbiUf'Z]Y`'XYf'XifW\gW\b]hh`]W\Y

 $5b[YVchgdfY]g`XYg`6Yn]f_g``Ui h`GhfYYh9Ugm`i a `) ž+`DfcnYbh`Ui Z), ("$$$`I G!8c``Ufž`XYb`b]YXf][ghYb`GhUbX`gY]h`&$%* "`8]Y \\ 5b[YVchgdfY]gY`]b`6fcc_`mb`V`]YVYb`i bj YfÅbXYfhž`k Å\fYbX`X]Y`]b A Ub\UhhUb`gh]Y[Yb"$

 $BY][\Vcf\ccX'< ci\ g]b[\ GYfj\]WY \czBYk\ Mcf_ 7]hmžY]bY\\ [Ya Y]bbØhn][\ Y \cdf[\ Ub]gUh]cbžX]Y \A YbgWXYb \a]hb]YXf][\ Ya \V]g\\ a]hhYfYa \ 9]b_ca a Yb ??fYX]hX]YbghY]ghi b[\ Yb ii bX 6]`Xi b[\ ini K \cbY][\ Ybhi a \ UbV]YhYhž\]`Zh]b `XYf`FY[\ Y``%$$ \V]g`%) $`A YbgWXYb 'A\f`]WX \VY]a \ ? Ui ZY]bYg`9][\ Yb\Y]a gž`gU[\ hXYf \ J cfghUbXgj cfg]hnYbXY \ 8 Yff]_; f][\ [g''=a \ \\\\\\ Y\bX\XYf`DUbXYa]Y \\\\``Ui Z'k Yb][\ Yf`U`g`&) ž`XU`?i bXYb`k \A\fYbX`XYf`DUbXYa]Y \\\``Ui Z'k Yb][\ Yf`CfYb`i bX`?fYX]hY]bVi \ Yb`\]bbY\a Yb`a i gghYb''$

±b 'E i YYbg'_Ui ZYb '±bj YghcfYb' < Ài gYf']b 'J]YfhY `b 'a]h 'b]YXf][Ya V]g'a]hh`YfYa '9]b_ca a Yb 'i bX'j YfXfÀb[Yb'; Ya Y]bXYbž']bXYa g]Y ' < Ui gVYg]hnYfb 'gi [[Yf]YfYbž'XUgg'XYf'J Yf_Ui Z'XYf'Y]bn][Y K Y['Z]bUbn]Y```]ghž'gU[h'McgY`]b'; YbUc!9ghfY``Už'; YgW\ÀZhgZØ\fYf]b j cb 'BY][\Vcf\ccX' < ci g]b['GYfj]WYg']a '6Yn]f_"

<Ui gVYg]hnYf']a 'GhUXhhY]`'5ghcf]U']b'Ei YYbg'\UVYb'_Øfn`]W\']\fYa
5V[YcfXbYhYb']a 'GhUUhgfUhž'Nc\fUb'? "'A Ua XUb]ž'YI d`cX]YfYbXY
9bYf[]Y!'i bX'J Yfgcf[i b[gfYW\bi b[Yb'[Ya Y`XYhž'X]Y' X]Y
5ghcf]UbYf'XUni 'nk]b[Ybž'g]W\'ni 'ZfU[Ybž'cV'g]Y'Yg'g]W\'`Y]ghYb
_`ObbYbž']b']\fYb' < Ài gYfb'i bX']b'i bgYfYf'BUW\VUfgW\UZh'ni
V`Y]VYb ž'qU[h'Yf"</pre>

 $8]Y `BYk `Mcf_Yf `XYf `5fVY]hYf_`UggY `_A`a dZYb `gY]h `$J \fnY \bhYb `a]h XYf `A Uf_ha UW h `XYf `; Ybhf]Z]n]Yfi b[ž `X]Y `g]Y `Ui g `] \fYb `< A`i gYfb$

 $Xf\grave{a}b[h"`9g`_\grave{o}bbhY`gWk]Yf][Yf`kYfXYbž`Ub`]\bYb`ZYghni\U`hYbž`XUX]YgYf`dUbXYa]YVYX]b[hY`k]fhgWkUZh`]WkY`8fiW_`Ub\\`a`h"$

GhYZZ:7\½;Yn']b'Y]bYa '?cffYgdcbXYbhYb'jcb': H'7\]WU[c

EiY``Y.:]bUbW]U``H]a Yg

8 YhU]`g

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. '_fmd hc!b Yk g"Uh