

?fmdhc VfYbbYb ``UggYb `p`:]bUbnnY]hYb

GhYd\Yb'7YWW\Yhh]']gh'FcgYb': Ua]`m'7\U]f']b'\bhYfbUh]cbU`
:]bUbW'Ub'XYf'6fUbXY]g'\bhYfbUh]cbU`'6i g]bYgg'GW\cc`''?]a
GW\cYb\c`hn']gh'Ya Yf]h]YfhYf'_`]b]gW\Yf'DfcZYggcf'Ub'XYf
GhYfb'GW\cc`'cZ'6i g]bYgg'XYf'BM "'BUW\XYa
Ni gUa a YbVfi W\'j cb': HL'gc``hYb'X]Y'6Y\ÒfXYb'XYa '8fUb[
k]XYfghY\Ybz'Y]bYb'dUfU``Y`Yb'FYW\hg!'i bX
FY[i`]Yfi b[gfU\a Yb'ZØf'X]Y'?fmdhc]bXi ghf]Y'ni 'gW\UZZYb'''9g
]gh'j]Y``VYggYfz'b]W\hg'ni 'hi b'i bX'?fmdhc'Y]bZUW\'VfYbbYb'ni
`UggYb'''9]b'U_h]j Yg'9]b[fY]ZYb'k ØfXY'Y]bYa 'GmghYa
i bj YfX]YbhY'@Y[]h]a]h\h'j Yf'Y]\Ybz'XUg'k Yb]['ni f
I bhYfghØhni b['YW\hYf'k]fhgW\UZh`]W\Yf'5_h]j]h\h\b'b'VY]hf\[A\[h''
9g'k ØfXY'Ui W\'Y]b'cZZ]n]Y``Yg'; ØhYg]Y[Y``ZØf'Y]b'GmghYa
XUfghY``Ybz'XUg'XYfnY]h'_Y]bY'6YXfc\i b['ZØf'X]Y
:]bUbnghUV]`]h\h'XUfghY``hz'i bX'ni '/ Ua d\'Y``]d/

 $GhYd \ Yb \ 7YWWX \ Yhh] \]gh \ FcgYb \ : Ua \]`m \ 7 \ U]f \]b \ \Rightarrow hYfb \ Uh]cb \ U`$ $:]b \ Ub \ WY \ Ub \ XYf \ 6f \ Ub \ XYf \ b \ Uh]cb \ U` \ 6i \ g]b \ Ygg \ GWX \ cc```?]a$ $GWX \ cYb \ c`hn' \]gh \ Ya \ Yf]h]YfhYf' _`]b \]gWX \ Yf' \ Dfc \ ZYggcf' \ Ub' \ XYf' \ GhYfb \ GWX \ cc``c \ Z'6i \ g]b \ Ygg' \ XYf' \ BM \ "`$

BUWX 'XYa 'Ni gUa a YbVfi WX 'j cb': HL 'gc``hYb'X]Y '6Y\ÒfXYb'XYa 8fUb['k]XYfghY\Ybž'Y]bYb'dUfU``Y`Yb'FYWX hg!'i bX FY[i`]Yfi b[gfU\a Yb'ZØf'X]Y'?fmdhc]bXi ghf]Y'ni 'gWXUZZYb"'9g']gh j]Y``VYgqYfž'b]WX hg'ni 'hi b'i bX'?fmdhc'Y]bZUWX 'VfYbbYb'ni '`UgqYb"'

9]b 'U_h]j Yg '9]b[fY]ZYb 'k ØfXY 'Y]b Ya 'GmghYa 'i bj YfX]YbhY @Y[]h]a]hÀh'j Yf 'Y]\Ybž'XUg 'k Yb]['ni f'I bhYfghØhni b['YW\hYf k]fhgW\UZh`]W\Yf '5_h]j]hÀhYb 'VY]hfÀ[h'' '9g 'k ØfXY 'Ui W\ 'Y]b 'c ZZ]n]Y ``Yg ; ØhYg]Y[Y``ZØf 'Y]b 'GmghYa 'XUfghY``Ybž'XUg 'XYfnY]h'_Y]bY 6YXfc\i b['ZØf'X]Y :]bUbnghUV]`]hÀh'XUfghY``hž'i bX'ni ': cfXYfi b[Yb bUW\ 'ÒZZYbh`]W\Yb 'FYhhi b[gU_h]cbYb 'ZØ\fYbž'k Ybb'?fmdhc

6Y]::]bUbnYb:XfY\hig]WX:U``Ygii a 'J YfhfUi Yb":8Yf
J YfhfUi Ybgj Yf`i gh'Xi fWX:ni bY\a YbXY:5i gZÀ``Y'ZØ\fh'VYfY]hgini a
I bhYf[Ub['j cb'?fmdhc":8]Y'A Uf_h_Ud]hU`]g]Yfi b['XYf'i bnÀ\`][Yb
A ØbnYb ']gh'gY]h']\fYa '< OWXghghUbX']a 'Bcj Ya VYf'&\$&%'i a
Yhk U'+) 'DfcnYbh'[Ygi b_Yb"

?fmdhc!A Uf_h_Ud]hU`]g]Yfi b[`ff` hb\tž`%"`\Jbi Uf`&\\$\+`V]g`% " Bcj Ya VYf`&\\$\&`| `Ei Y``Y. `7c]ba Yhf]Wg"`6YUW\hYb`G]Y\text{z`XUgg`X]Y B]j YUi g`\]Yf`Yhk Ug`b]YXf][Yf`g]bX`U`g`X]Y`j cb`7c]bA Uf_Yh7Ud"

9gʻ]ghʻgWk Yfʻj cfghY``VUfžʻXUggʻg]WkʻXUgʻJ YfhfUi Ybʻ]bʻ?fmdhcʻj cb XYaʻ5i ga U» i bXʻl a ZUb[ʻXYfʻ5i gZÀ``Yʻj cbʻ: HLʻYf\c`h"ʻ6]gʻj cf _i fnYaʻk Ufʻ: HLʻY]bYʻZØ\fYbXYʻ6ÒfgYʻi bXʻk i fXYʻk Y]h\]bʻUʻg F]WkhgWk bi fʻ]bʻY]bYfʻ6fUbWkYʻj c``YfʻGWkUf`UhUbYʻUb[Ydf]YgYb'' : HLʻ\Uhʻg]WkʻYXcWkʻVYk i gghʻZØfʻY]bYʻ5bg]YX`i b[ʻ]bʻY]bYf ; Yf]WkhgVUf_Y]hʻYbhgWk]YXYbžʻX]YʻUi »Yf\U`VʻXYgʻfYWkh`]WkYbʻi bX fY[i`Uhcf]gWkYbʻNi ghÀbX][_Y]hgVYfY]WkgʻXYfʻBUh]cbYbʻa]hʻXYb [fÒ» hYbʻ:]bUbngmghYa Ybʻ`]Y[h''

8 UfØVYf`\]bUi g`nY][Yb`6Yf]W\hY`Yhnhž`XUgg`Yg`: HL`Ub HfUbgdUfYbn`a Ub[Y`hYž'?i bXYb[Y`XYf`a]ggVfUi W\h`k i fXYbž`Ub ; YgW\ÀZhYb`a]h'j YfVi bXYbYb`DUfhY]Yb`VYhY]`][h'k UfYbž`Y]bY gW\k UW\Y`I bhYfbY\a YbgZØ\fi b[`\UhhYb`i bX D\Ubhca g]W\Yf\Y]hYb`gck]Y`UbXYfY`i bg]W\YfY`DfU_h]_Yb U_nYdh]YfhYb"

9]bZUWX [YgU[hž:XUg:?fmdhcgmghYa ž:k]Y:Yg:XYfnY]h:YI]gh]Yfhž:]ghb]WXh:bUWX\U:h]["C\bY:_`UfY:ibX:`Y]WXh:XifWXgYhnVUfY
9][YbhiagfYWXhY:kUf:Yg:b]Y:g]WXYf:ibX:YZZY_h]jž:g]WXUiggWX:]Y»`]WX:UiZ'df]jUhY:=bjYghcfYb:ni:jYf:UggYbž:ia:XUgJYf\U:hYb:ibXifWXg]WXh][Yf:JYfa]hh:Yf:ni:ØVYfkUWXYb:ibX:niX]gn]d`]b]YfYb":9g:[]Vh:_Y]bY:5igg]WXh:UiZ:Y]bY:hYWXbc`c[]gWXY@Ògib[:X]YgYf:ifU:hYb:DfcV:YaY"

8]Y'[fc»Y': fU[Y']gh'U`gcžcV'X]Y'6Y\ÒfXYb'Y]bYb'bYi Yb
FY[i`]Yfi b[g!'i bX'5i Zg]W\hgfU\a Yb'gW\UZZYb'gc``hYbž'XYf
9][Ybhi a gfYW\hY'gW\Øhnh'i bX'X]Y'; fi bXgÀhnY'XYf'G]W\Yf\Y]h'i bX
Gc`]X]hÀh'Xi fW\gYhnh''6Ygcf[h'ØVYf'k Y]hYfY'JYf`i ghY'Xi fW\XYb
Ni gUa a YbVfi W\'j cb'?fmdhc'ZcfXYfb'j]Y`Y'A YbgW\Yb'bYi Y
FY[Y`b'ni a 'GW\i hn'XYf'JYfVfUi W\Yf''

fcb]gWXYfkY]gY'kYfXYb'JYfgi WXYž'Y]bY'gYdUfUhY'Ghfi_hif'nif FY[i`]Yfib['ibX', VYfkUWXib['jcb'?fmdhc'ni'gWXUZZYbž'YXcWX bif'XUg':]bUbngmghYa 'YffY]WXYb'kYb][Yfž'b]WXh'aY\fž'g]WXYf"

8]Yg'[]`h'Ui g'nk Y]'; fØbXYb"'9fghYbg'k]fX'Yg'X]Y'6Ub_Yb'XUni
Yfa i h][Ybž'gck c\`'?fmdhc!5ggYhg'ni '_Ui ZYb'U`g'Ui W\'U`g
G]W\Yf\Y]h'[Y[Yb'g]Y'ni 'j Yf`Y]\Ybž'k cXi fW\'XUg'6Ub_YbgmghYa
UbZ\``]['ZØf'g]b_YbXY'A Uf_hk YfhY'k]fX"'=a'; Y[YbgUhn'XUni '\UhhY
gY`Vgh'XYf'Ub\U`hYbXY'Ni gUa a YbVfi W\'j cb'?fmdhck YfhYb'i bX
!]bgh]hi h]cbYb'dfU_h]gW\'_Y]bY'5i gk]f_i b[Yb'Ui Z'XUg
K c\`Yf[Y\Yb'XYf'hfUX]h]cbY``Yb':]bUbna Af_hY'i bX
!i bhYfbY\a Yb"

Nk Y]hYbg'k ØfXYb'bYi Y'FY[Y'b'ni Y]bYf'A][fUh]cb'XYf
:]bUbnhÀh][_Y]h'j cb'XYf'hfUX]h]cbY``Yb':]bUbnk Y`h']b'X]Y'bcW
k Yb][Yf'fY[i']YfhYž'UVYf'bYi 'gUb_h]cb]YfhY'?fmdhck Y`h'ZØ\fYb"
Gck c\``?fmdhc'U`g'Ui W\'hfUX]h]cbY``Y':]bUbnYb'g]bX'Y]bZUW\
?ca V]bUh]cbYb'Ui g'Y]bYf'8 UhYbVUb_i bX'Y]bYa '7ca di hYfWcXY"
9g'k ÀfY'ZØf'Y]bY'; fi ddY'j cb'HYW\b]_Yfb'Y]bZUW\ž'Y]bYb
VY`]YV][Yb'GUhn'VYX]b[hYf'7Ug\Zck g'j cb'Y]bYa ']b'Xyb'UbXYfYb
i a ni k UbXY`b"'GhY``Yb'G]Y'g]W\'ni a '6Y]gd]Y`'j cfž'Ya UbX
YbhgW\Y]XYh'g]W\'XUZØfž': cfXYfi b[Yb'Ub'gY]bY':]fa U'U`g'?fmdhc!
Hc_Yb'ghUhh'U`g'\Yf_Òa a `]W\Yg'9][Yb_Ud]hU`'Ui gni [YVYbž'i a 'j cb
`cW_YfYfYb'FY[Y`b'ZØf'CZZYb`Y[i b[ž'6i W\\U`hi b[ž'JYfk U\fi b[i bX'XYf[`Y]]W\Yb'ni 'dfcZ]h]YfYb"

 $\label{thm:continuity:continui$

 $6 \ \mathsf{Ub_Yb} \ \check{\mathsf{z}}' < \grave{\mathsf{A}} \ \mathsf{b} \ \mathsf{X}' \mathsf{Y} \ \mathsf{f} \ \check{\mathsf{z}}' \mathsf{J} \ \mathsf{Y} \ \mathsf{f} \ \mathsf{g} \mathsf{g} \\ \mathsf{b}'' \ \mathsf{b} \ \mathsf{b} \ \mathsf{b}'' \ \mathsf{b} \$

?fmdhcg'b]Wkh'_Ui ZYb'i bX`\U`hYb'cXYf'U`g'G]WkYf\Y]h'U_nYdh]YfYb XØfZYb"'?fmdhc']gh'\Yi hY'[fÒ»hYbhY]`g'bi f'Y]b'A i `h]d`UmYf!Cb`]bY! J]XYcgd]Y`'fk]Y'K cf`X'cZ'K UfWfUZhŁ"'K Ybb'dfU_h]gWk'U``Y HfUbgU_h]cbYb']bbYf\U`V'XYf'?fmdhckY`h'c\bY'JYfV]bXi b[Yb'ni f FYU`k]fhgWkUZh'V`Y]VYbž'_ÒbbhY'XYf'DfcnYgg'[YbUi gc'[i h'Ui Z'XYa A Ufg'ghUhhZ]bXYb'i bX'X]Y'hfUX]h]cbY``Y':]bUbnkY`h'i bVYfØ\fh `UggYb"

8 UgʻØVYf[YcfXbYhYʻN]Y``XYfʻdc`]h]gWXYbʻ9bhgWXY]Xi b[ghfÀ[Yfgc``hYʻYgʻgY]bžʻ?fmdhcʻgmghYa]gWX·]ffY`Yj Ubhʻni '\U`hYb"'8YfʻVYghY K Y[ž'X]Ygʻni 'hi bžʻVYghY\hʻXUf]bžʻYgʻi bhYfʻXYa '8 fi W_ʻgY]bYf i bg]WXYfYbʻi bXʻi bgc`]XYbʻ; YgWXÀZhgdfU_h]_Ybʻ]a d`cX]YfYbʻni `UggYb"'=bʻXYfʻNk]gWXYbnY]hʻgc``hYbʻX]Yʻ6Y\ÒfXYbʻghÀbX]['Ui ZʻX]Y 5 i ZhY]WXbi b[Ybʻ\]bk Y]gYbžʻXUggʻ?fmdhcʻj c``Yfʻ: Y\`Yfʻi bXʻ6Yhfi []gh"

5 b g h U h Y b 'b Y i Y b 'f Y W h '] W Y b 'i b X 'f Y [i `U h c f] g W Y b 'F U \ a Y b 'n i g W U Z Z Y b ž 'X Y f '? f m d h c ``Y [] h] a] Y f h ž 'g c ``h Y b 'k] f 'Y g 'Y] b Z U W 'V f Y b b Y b `Ugg Y b "

EiY``Y.:]bUbW]U``H]a Yg

8 YhU]`g

6Ygi WXYb G]Y i bg Ui Z _fmdhc!bYk g"Uh