

8]Y'B: $H!A \hat{A}f_hY'Yf^YVhYb']a \rightarrow b]'Y]bYb$ A cbUh'XYf'5W hYfVU bZU fh

8]Y'?i bgh!B: Hg'XYf'K UbXYfYfg'j YfnY]W\bYhYb'nk]gW\Yb XYa') "'i bX'*"'\delta']Y]bYb'Ua 'gh\Af_ghYb'[Ygh]Y[YbYb
A]bXYghdfY]gž'k \A\fYbX'D]I Y''J Ui 'hg'Di b_g'7ca]Wa Y\f'U'g
YXY'UbXYfY'Hcd!GUa a 'i b['Z]Y''9h\YfYi a 'BUa Y'GYfj]]W!
8ca U]bg'j YfnY]W\bYhYb'5bZUb['X]YgYf'K cW\Y'XUg'[f\Omega') hY
9H<!J c'i a Yb'Ui Z'CdYbGYU'CdYbGYU\z'XYf'[f\Omega') hY'B: H!
A Uf_hd'Uhn\z'Yfn]Y'hY']a '\delta'b]Y]b'< UbXY'\gj c'i a Yb'j cb'* (*\z'*
A]``]cbYb'I G!8c'`Uf\z'k Ug'Y]bYa 'F\OmegaW_[Ub['j cb'a Y\f'U'\g'*)
\['[Y[Yb\Omega']Ybhgdf]W\h''[Ya \A\name ni '8 UddFUXUf''Gc`UbU
B: H'fbcb!\z' b[]V'Y'hc_Yb\z'A Uf_hd`Uhn'A U[]W9XYb
9]b[YhfU[Yb'ZUgh'\%\cancel{B}'A]``]cbYb'I G!8c'`Uf'< UbXY'\gj c'i a Yb
]a '`YhnhYb'A cbUh\z'Y]b'F\Omega'[Ub['j cb'*'\z') 'i 'gY]h'A U]"'8]YgY
8 UhYb'XYi hYb'XUfUi Z'\]b\z'XUgg'g]b_YbXY'DfY]gY'j]Y'Yf
\[ni [fi bXY'']Y[YbXYf'/ Ua d\A\Y'`]d/

- 8]Y?i bgh!B: Hg'XYf'K UbXYfYfg'j YfnY]W\bYhYb'nk]gW\YbXYa') "i bX'* "'*\`]'Y]bYb'Ua 'gh\Af_ghYb'[Ygh]Y[YbYbA]bXYghdfY]g\(\tilde{z}\)'k \(\tilde{A}\)fYbX'D]I Y`'J Ui `hg'Di b_g'7ca]Wa Y\fU\`g'\YXY'UbXYfY'Hcd!GUa a `i b['\tilde{Z}\)'
- 9h\YfYi a 'BUa Y'GYfj]WY!8ca U]bg'j YfnY]WXbYhYb'5bZUb[X]YgYf'K cWXY'XUg'[fÒ» hY'9H<!Jc`i a Yb'Ui Z'CdYbGYU

CdYbGYUžXYf'[fÒ» hY'B: H!A Uf_hd`Uhnž'Yfn]Y`hY']a '* | b]'Y]b < UbXY`gj c`i a Yb'j cb'* (* ž* 'A]``]cbYb'l G!8c``Ufž'k Ug'Y]bYa FØW_[Ub['j cb'a Y\f'U`g'*) 'i '[Y[YbØVYf'A U]'Ybhgdf]W\h"'[Ya À» ni '8 UddFUXUf"

Gc`UbU'B: H'fbcb!Z b[]V`Y'hc_YbŁ'A Uf_hd`Uhn'A U[]W9XYb' 9]b[YhfU[Yb'ZUgh'%%%'A]``]cbYb'l G!8c``Uf'<UbXY`gj c`i a Yb']a `YhnhYb'A cbUhž'Y]b'FØW_[Ub['j cb'*' ž)'ı 'gY]h'A U]"

8]YgY'8UhYb'XYi hYb'XUfUi Z'\]bž'XUgg'g]b_YbXY'DfY]gY'j]Y`Yf ni [fi bXY'`]Y[YbXYf'?fmdhck À\fi b[Ybž'Y]bgW\`]Y»`]W\'9h\Yf'f9H<Ł i bX'Gc`UbUž']b'?ca V]bUh]cb'a]h'g]b_YbXYb'B: H!: `ccfg'ni 'Y]bYa 9]bVfi W\'XYg'<UbXY`gj c`i a Ybg'VY][YhfU[Yb'\UVYb'_ObbhYb"

 $\label{thm:linear} HfYbXg`]b`XYb``YhhhYb`gYWXg`A cbUhYb`nY][Ybž`XUgg`X]Y`B: H! \\ 6cXYbdfY]gY`j]Y``k U\fgWXY]b`]WXYf`[Ygh]Y[Yb`g]bXž`k Ybb`X]Y`9H< ghY][hž`[Ya Å» `ni `8 UhYbž`X]Y`j ca `B: H!5bU`mghYb`4 di b_-$) - ni gUa a Yb[YghY``h`k i fXYb"$

EiY``Y.'4 dib_-\$) - 'Ui Z'Hk]hhYf'

8 Ug' < UbXY`gj c`i a Yb`XUfZ'YXcW\`b]W\h'j c``ghÀbX]['a]h`XYb DfY]gYb'j Yfk YW\gY`h'k YfXYbž'gc'6`cca VYf['[Ya Y`XYh'Ui Z[fi bX g]b_YbXYf'l a gÀhnY' Y]b[YVfcW\Yb 'ni 'gY]b" J]Y`Y`: U_hcfYb`hfU[Yb`ni f`6YkY[ib[`XYf`B: H!A]bXYghdfY]gY`VY]"
5b[Yg]Wkhg`XYf`HUhgUWkYž`XUgg`B: Hg`A ca Ybhi a!5b`U[Yb`g]bXž`]gh
Yg`k U\fgWkY]b`]WkYfž`XUgg`X]Y`6cXYb_i fgY`\Yi hY`ghY][Ybž`kYbb
g]Y`[YghYfb`[Ygh]Y[Yb`g]bXž`i bX`i a[Y_Y\fh`[Yhk]hhYfh
4 di b_-\$)-"

8 Yf [YgUa hY 'A Uf_hk Yfh XYg 'B : H! 2 _cgmghYa g 'gh]Y[']a 'nk Y]hYb E i UfhU` XYg '\\ fYg 'j cb'') z* 'A]``]UfXYb 'I G!8c``Uf 'Ui Z'&&z'
A]``]UfXYb 'I G!8c``Uf '' k]Y 'X]Y '8 UhYb 'nY][Yb 'j cb 'B : H; c"
8 Ybbc W\ 'nY][h'X]Y 'Zc`[YbXY '; fUZ]_z XUgg 'X]Y 'NU\``XYf 'B : H!
\[\dag{\text{

EiY``Y. Bcb: i b[]V`Y'

±b·6Yni [ˈUi Z'6`i Y!7\]d!B: Hgˈk]YgʻF YVcʻXUfUi Z'\]bžʻXUggʻX]Y A]bXYghdfY]gYʻ5bZUb[ˈ⅓ b]ˈa Ò[`]WXYfk Y]gYʻ[Ygi b_Ybʻg]bXžʻUVYf Y]bYbʻK]YXYfUi ZgWki b[ˈYf`YVhYbžʻXYfʻa]hʻXYfʻB M7 "B: H! ?cbj Ybh]cbʻni gUa a YbZ]Y`"

8 UddF UXYf`ghY``hY`ZYghž`XUgg`A YYV]hg`a]h`Y]bYa `Gdfi b[`j cb`+*

1 ž`Ub[Y[YVYb`]b`9H< ž`XUg`: Y`X`UbZØ\fhYž`k À\fYbX`8ccX`Yg`i bX

7 fmd hcDi b_g`((`i `i bX`('`i `Yf\c`hYb`i bX`6cfYX`5dY`MUW\h`7`i V
&'`i `ni `Y[hY"\8]Y`NU\``XYf`7 fmd hcDi b_g!< ÀbX`Yf`i bX`!J Yf_Ài ZY
gh]Y[`Ua `%"`¾`]`YVYbZU``g`i a `, +`i `Vnk "`%&\$`i "

9g']gh'k]Wkh]['ni '\]bhYfZfU[Ybž'cV'XYf'A]bXYghdfY]g'cXYf'XYf
8i fWkgWkb]hhgdfY]g'ZU]fY'\dank X]_UhcfYb'ZØf'XYb'k U\fYb'K Yfh'Y]bYg
B: H!Dfc'Y_hg'g]bX"'8Yf'A]bXYghdfY]g'a U['XYf'bYi YghY'DfY]g'XYg
[Øbgh][ghYb'B: H']b'X]YgYf'GUa a `i b['gY]bž'UVYf'Yf']gh'bi f'Y]bYf
j cb'j]Y'Yb'K Yfh]bX]_UhcfYb'k]Y'GY`hYb\Y]hž'BØhn`]Wk_Y]hž
6Yg]hn[YgWk]WkhYž'Fi Z'XYg'GWk\OdZYfg'i bX'?Ài ZYf]bhYfYggY"'8]Y
?Ài ZYfbUWk\ZfU[Y'fbXYf'XYfYb': Y\`YbŁž'X]Y'ni 'Y]bYa '5bgh]Y['cXYf

 $F @W_{_}[Ub["XYf"5bnU\"XYf"B: H!? \`{A}i ZY"Z@\fhz"Z@\fh'ni GW_{_}k Ub_{_}i b[Yb"XYf"A]bXYghdfY]gY""$

8]Y'JYfÀbXYfi b[Yb']b'XYf'5bnU\`XYf'hÀ[`]W\Yb'B: H!JYf_Ài ZYg]bX'nk Uf'gY]h'5bZUb['&\$&&'fØW_`Ài Z][ž']b'XYb'`YhnhYb'XfY]
A cbUhYb'YXcW\'b]W\h'gc'XfUgh]gW\"'Bcb: i b[]V`Y!8UhYb'nY][YbžXUgg'X]Y'NU\`XYf'hÀ[`]W\Yb'JYf_Ài ZY'9bXY'¾b]'Ui Z'&\$"\$\$\$
[Ygi b_Yb']ghž'A]hhY'5df]``YXcW\'a]h'%) "\$\$\$'X]Y'b]YXf][ghYJYf_Ui ZgnU\`XYg'j Yf[Ub[YbYb'¾\\fYg'j YfnY]W\bYh'k i fXY"

5bnU\``XYf'hÀ[`]W\Yb'B: H!JYf_Ài ZY'j cb'5df]``V]g'⇒`]'&\$&&'p EiY``Y.'Bcb:ib[]V`Y'

5i hc[`md\g'i bX'5fh6`cW_g'[Y\OfhYb'ni 'XYb'Ua 'VYghYb
j YfX]YbYbXYb'GUa a `i b[Yb']a '?i bghgY[a Ybhžk À\fYbX'65M7g
Ch\Yfg]XY!@UbX'XYf'VYghY'B: H!JYfX]YbYf']a 'A YhUj Yfgi a 'k Uf
i bX'=b!; Ua Y!B: Hg'j cb'51]Y'=bZ]b]hm'i bX'@cch'X]Y'\OW\ghYb
I a gÀhnY'Yfn]Y`hYb"'

I h]`]hm`B: Hg`k]Y`X]Y`XYnYbhfU`Yb`8ca U]bg`XYg`9h\YfYi a `BUa Y GYfj]WV`fPBGŁ'j YfnY]W\bYhYb`]a `\dim\b]'Y]b`Yf\O\hYg`\dhYfYggYž k cVY]`XYf`Hcd!GY``Yf` dcfbc"Yh\ `ZØf`%, (`9H<`cXYf`&\$' ", -&`` []b["'' "'\dot\angle)]Yfh`XYf`JYf_Ui Z`XYg`hYi YfghYb`9BG!BUa Ybg \$\$\$"Yh\ `ZØf' \$\$`9H<`cXYf'\ &\$"\$\$\$`I G!8c``Uf"'`

5bXYfY'VYa Yf_Ybgk YfhY'¾ b]!; Yk]bbYf'g]bX'XUg'; cV`]bHck b!
Dfc'Y_hž'XUg' YhUV`]YfhY'DfU_h]_Yb'j Ò``]['ØVYf'XYb'<Ui ZYb
[Yk cfZYb'\Uh ž']bXYa 'Yg'X]Y'A ØbnY'ZfY]'[Ya UW\h'\Uh'i bX'k YXYf
Y]bYb'8]gWcfX'bcW\Y]bY'FcUXa Ud'\Uhž'gU[hY'FYVc"

 $8 \ Yf'; \ U``Yfm'c Z'8][]hU``5ggYhg'fl \ C 8 5 L'A]bh'DUgg'_`YhhYfhY']b'X]Y \\ 7 \ Ufhgž'k \ Y]``Yf'B: H! $\UVYfb'Ni [Ub['ni 'df]j Uh'_i fUh]YfhYb$

GUa a `i b[Yb'a cXYfbYf'?i bgh'[Yk À\fh'i bX'Ui ZghfYVYbXY'B: H!?@bgh`Yf'i bhYfgh@hnh"

#bg[YgUa h'⅓ b]'\Yfj cf[Y\cVYb 'Y]bY']a a YbgY'Jc`Uh]`]hÀh'i bX
5_h]j]hÀh']bbYf\U`V'XYf'?fmdhc]bXi ghf]Y"'I bX']bbYf\U`V'XYg'K YV'!
i bX'B: H!6YfY]W\g'k YfXYb'bi f'X]Y'GUa a `i b[Ybž'X]Y'XYb
j YfgdfcW\YbYb'K Yfh'`]YZYfb'_ÒbbYbž'Yg'gW\UZZYb"'5i g[Y\YbX'j cb
a Y\fYfYb'A]``]cbYb'8c``Uf'Ub':]bUbna]hhY`b'i bX'5_h]j]Yfi b[Yb
VY]'B: H"BM7'a ØggYb'K YV'!I bhYfbY\a Yb'VYggYfY'K Y[Y'Z]bXYbž
i a 'X]Y'@cmU`]hÀh']\fYf'?i bXYb'ni 'a YggYb"

 $\label{eq:c.yb} $$ < c.Yb.G]Y.g]W.^YXYb.5VYbX.X]Y.VYghYb.?fmd.hc! $$ BYi][_Y]hYb.i bX.9f_Ybbhb]ggY.XYg.HU[Yg.]b.=&fYbDc.ghY]b[Ub["." $$$

8]Y 'Dcgh' B: H!A Af_hY 'Yf`YVhYb']a ' \Rightarrow b]'Y]bYb'5W\hYfVU\ba cbUh]gh'_Y]bY'Z]bUbn]Y``Y'6YfUhi b["

8YhU]`g

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. '_fmd hc!bYk g"Uh